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Multi-Period MIP Models for Real-Time Scheduling and Resource Allocation in Digital Twin-Enabled Systems

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ABSTRACT

The use of Digital Twin (DT) technology in advanced industrial systems enables us to have better real-time monitoring, control, and optimization of complex processes. This paper presents new multi-period mixed-integer programming (MIP) models that can solve the problems of real-time scheduling and resource allocation in Digital Twin-based environments. The new models include dynamic system behavior across a number of different time periods with resource capacity constraints, task precedence, and working deadlines. Taking advantage of the real-time data streams from Digital Twins, the models learn and adapt to system variations and uncertainty to facilitate optimal decision-making for resource utilization and production efficiency. Computational experimentation on conventional case studies demonstrates the practicability and scalability of the approach and its potential to increase responsiveness and operational efficiency in cyber-physical systems and smart manufacturing.

1. Introduction

Over the last few years, the advent of Digital Twin (DT) technology has changed the game of industrial automation and intelligent manufacturing. A Digital Twin refers to a virtual copy of a physical system that provides real-time monitoring, simulation, and optimization of processes continuously through sensor, machine, and control system data integration. This feature makes it possible for organizations to maintain greater visibility, predictive maintenance, and rapid decision-making in sophisticated and dynamic production environments.

Real-time scheduling and resource allocation are key components to ensure operational effectiveness and responsiveness in such systems. Traditional models of scheduling are not designed to capture the temporal and stochastic nature of modern production systems, especially when adjustments need to be addressed at high rates and uncertainties run constantly. Multi-period mixed-integer programming (MIP) models provide a solid mathematical framework to model these dynamic settings by considering multiple horizons and complex resource constraints.

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This research focuses on the formulation and analysis of multi-period MIP models with particular relevance to real-time scheduling and resource allocation in Digital Twin-based systems. Based on the rich, real-time data streams offered by Digital Twins, the developed models adjust scheduling decisions and resource allocations dynamically to enhance key performance measures such as throughput, utilization, and lead times. The merge of DT technology with advanced optimization models is a promising solution to bridge the gap between physical and cyber-physical systems and enhance more resilient, versatile, and intelligent manufacturing systems.

The remainder of the paper is organized as follows: Section 2 recapitulates previous studies on scheduling, resource allocation, and Digital Twin implementation. Section 3 outlines the formulation of the multi-period MIP model, assumptions, and constraints. Section 4 discusses computational experiments and results discussion. Section 5 summarizes the research and outlines avenues for future research.

2. Literature Survey

The integration of Digital Twin (DT) technologies with real-time resource planning and allocation is an emerging area of research for cyber-physical manufacturing systems. Multi-period Mixed-Integer Programming (MIP) models are increasingly used to support intelligent decision-making in dynamic and intricate environments. The survey consolidates recent findings and patterns in this direction by evaluating research in manufacturing, logistics, maintenance, and distributed systems. Digital twins offer virtual imitation of physical systems with real-time visibility and adaptive control. In their work, Li et al., [1] developed a DT-based rescheduling model of intermodal terminals using hybrid methods, which was shown to minimize delays as well as enhance the operational efficiency by significant margins. Similarly, Hosseini-Motlagh et al., [2] incorporated a digital twin with a mixed proactive-reactive model for the optimisation of human milk supply chains, making the application of DT in humanitarian logistics viable.

The use of DTs in mobile networks was investigated by Nardini and Stea [3], who made simulation services for 5G/B5G infrastructures possible with dynamic resource provisioning—a methodology translatable to production systems to make adaptive task scheduling possible. Wallrath et al., [4] suggested a time-decked MIP formulation for chemical batch plants in particular, resolving scheduling and lot-sizing under realistic constraints. Time-decomposed models are stepping stones for multi-period scheduling, especially if synchronized with DTs to capture the flow of time. Moreover, Yang et al., [5] applied risk-averse stochastic programming for dynamic lot sizing on reconfigurable assembly lines, demonstrating how decisions across periods could be treated under uncertainty. Meanwhile, Ye et al., [6] created a bi-level supply chain model under the cloud manufacturing, emphasizing the equilibrium between centralized planning and decentralized deployment, a key feature of DT-enabled architectures.

The integration of AI with scheduling has transformed DT resource management. Ghosh and Abawajy [7] used AI and reliability analysis to minimize concreting operations, while Pitakaso et al., [8,9] designed transformer-based models for energy-efficient tugboat scheduling and floating crane operations. The research exemplifies the growing trend of adopting AI into scheduling operations to handle variability and maximize task allocation. MajidiParast et al., [10] used graph convolutional networks for predictive railway maintenance, applying the DT principle of predictive modeling of system behavior. Likewise, Wang et al. [11] integrated UAVs and IoV to deploy servers, optimizing resource offloading with mobility and connectivity constraints. Gartner et al., [12] discussed work and product rotation in industrial house prefabrication with multi-mixed-model assembly lines benefiting from DT-based sequencing. Meanwhile, Dönmez [13] developed an optimization algorithm-based decision support system for airport ground operations with real-time scheduling in essential infrastructure.

In supply chains of semiconductors, Kumar et al., [14] emphasized reliability and disruption resilience—a key feature in dynamic resource scheduling in MIP models. Simulation-based scheduling was used by the study of Castiglione et al., [15] to advance flexibility exploration in Industry 4.0, supporting synergy between digital twin simulation and MIP solutions. Sheikh et al., [16] created a safe and intelligent CPS architecture, where real-time scheduling was one of the factors influencing infrastructure resilience. In alignment with such an environment, Lian et al., [17] created mobile robot scheduling under industrial CPS, employing a hierarchical mechanism for spatial-temporal coordination—an approach well suited for multi-period MIP solutions. Gong et al., [18] proposed multi-objective optimization for mobile assembly line job shop scheduling under dynamic electricity pricing, bridging operational scheduling and sustainability goals. Buckhorst et al., [19] developed a decentralized control strategy for mobile assembly lines, again highlighting the role of DTs in flexible line-less manufacturing. The evolution into sustainable and resilient systems is evident in some of the publications. Wang et al., [20] provided a general overview of green maritime logistics, while Sudan et al., [21] elaborated on supply chain resilience strategy through the application of lifecycle and disruption-mitigation perspectives. The findings assist in enhancing multi-period models through incorporating sustainability and uncertainty mitigation factors. Guarnaschelli et al., [22] gave a stochastic model for dairy supply chains with production and distribution planning. Yeni et al., [23] gave a lean and stochastic programming approach for aquaculture supply chains, with emphasis on the utility of integrated planning models.

Uncertainty scheduling was also discussed by Ma et al., [24] for aviation maintenance routing, acknowledging gaps in current models and the potential of new tech like DTs. Geurtsen et al., [25] examined multi-line maintenance scheduling, offering valuable inputs towards resource planning between distributed production facilities. Hamou et al., [26] conducted a survey on ML-based production scheduling, encouraging data-driven models capable of learning to adapt to changes in real-time—valuable in DT settings. Darchini-Tabrizi et al., [27] took the concept further by designing UAV-enabled MEC systems with intelligent task offloading, proving the potential of DT-driven edge systems for dynamic scheduling.

This literature evidences the convergence of digital twin technologies, AI, and MIP-based models in real-time scheduling and resource allocation. Promising directions are:

i. Combination of multi-agent DT platforms with decentralized optimization models.
ii. Incorporation of sustainability and resilience metrics in MIP formulations.
iii. Development of hybrid MIP-AI frameworks for predictive and adaptive scheduling.
iv. Extension of applications to smart logistics, intermodal terminals, and aerospace systems.

Despite such advancements, there has been considerable scarcity of multi-period MIP models that maximize the full, real-time information provided by Digital Twins to schedule and distribute resources. Most such works in the past rely on approximations for scheduling horizons or use heuristic methods that are not guaranteed to be optimal. This research aims to fill this void by developing robust multi-period MIP models that can receive real-time inputs from Digital Twins to optimize scheduling and resource allocation with multiple time horizons simultaneously.

3. Methodology

This subsubsection explains the MIP model formulation and solution approach for multi-period MIP models to optimize Digital Twin-aided real-time scheduling and resource allocation. The approach includes real-time feedback data from the Digital Twin system in dynamically modifying scheduling decisions over multiple time periods.

Let's establish the major sets, parameters, and variables that must be used to model and solve the multi-period scheduling and resource allocation problem.

Let $I = \{1,2,\dots,n\}$ denote the set of jobs/tasks, $R = \{1,2,\dots,m\}$ the set of resource types and $T = \{1,2,\dots,H\}$ the set of discrete time intervals within the planning horizon. For each task $i \in I$, its processing time is given as $p_i \in \mathbb{Z}^+$and its due date is defined as $d_i \in T$. At each time period $t \in T$, the available capacity of resource $r \in R$ is represented by $c_{r,t} \in \mathbb{R}^+$ and the usage of resource r by task i per unit time is given by $a_{i,r} \in \mathbb{R}^+$. The precedence relations among tasks are modeled using the set $\mathcal{P} \subseteq I \times I$ where $(i,j) \in \mathcal{P}$ implies that task i must be completed before task j starts. The binary decision variable $x_{i,t} \in \{0,1\}$ indicates whether task i starts at time t and $C_i \in T$ represents the completion time of task i.

A schedule $\mathbf{x} = \{x_{i,t}\}$ is said to be feasible if it satisfies several constraints. First, each task must start exactly once, as formulated in Eq. (1):

$$\sum_{t=1}^{H} x_{i,t} = 1 \tag{1}$$

Second, at any time period, the total resource usage must not exceed the available capacity. This requirement is captured by Eq. (2):

$$\sum_{i\in I} a_{i,r} \cdot \sum_{\tau=\max(1,t-p_i+1)}^{t} x_{i,\tau} \leq c_{r,t} \tag{2}$$

Third, precedence constraints must be respected so that a task cannot start before all its predecessors have completed. This is formulated in Eq. (3):

$$C_i \leq \sum_{t=1}^{H} t \cdot x_{j,t}, \quad \forall (i,j) \in \mathcal{P} \tag{3}$$

Lemma 1 (Validity of Task Completion Time) states that if each task $i \in I$ is scheduled to start exactly once at time s_i then the completion time is given by Eq. (4):

$$C_i = s_i + p_i - 1 \text{ where } s_i = \sum_{t=1}^{H} t \cdot x_{i,t}. \tag{4}$$

The lemma follows directly from the assumption that $x_{i,t} = 1$ only at time $t = s_i$, meaning the task starts at s_i and runs non-preemptively for p_i periods. Thus, it completes at $s_i + p_i - 1$. This result implies that completion times C_i can be derived directly from start times s_i and processing duration p_i which enables a linear representation in a mixed-integer programming (MIP) model.

Property 1 (Resource Feasibility) further requires that the execution of tasks must not violate resource constraints. Let $y_{i,t} = 1$ if task i is being executed at time t and 0 otherwise. The feasibility condition is expressed in Eq. (5):

$$\sum_{i\in I} a_{i,r} \cdot y_{i,t} \leq c_{r,t}, \quad \forall r \in R, \forall t \in T \tag{5}$$

3.1 Problem Description

The modeled system is a set of jobs or tasks that must be allocated to available but restricted resources (e.g., material, labor, equipment) over a planning period divided into discrete time intervals. Jobs have some processing times, precedence, resource requirements, and due dates. A schedule and resource assignment are to be determined to minimize total operating costs, tardiness, or maximize system throughput under capacity and operating constraints. This is a multi-period Mixed-Integer Programming (MIP) model for Digital Twin-supported system scheduling and resource allocation. The model integrates real-time updates of data within planning horizon T, with task scheduling, resource availability, and precedence considered.

The planning horizon is segmented into discrete time points indexed by $t = \{1,2,\dots,T\}$. Throughout this horizon, various task attributes—such as processing durations, release times, and resource availabilities—may change dynamically or be updated in real time based on feedback from a Digital Twin system. All resources involved are constrained by finite capacities, and these capacities must not be exceeded in any time interval. Furthermore, tasks are assumed to be non-preemptive, meaning once a task is started, it must be executed to completion without interruption. Precedence constraints between tasks must also be strictly respected to ensure logical sequencing. To formally describe the problem, several sets and indices are defined: $I = \{1,2,\dots,n\}$ denotes the set of jobs or tasks; $R = \{1,2,\dots,m\}$ represents the set of resource types; and $T = \{1,2,\dots,H\}$ corresponds to the set of discrete time periods over the planning horizon. Each task has an associated processing duration p_i, a due date d_i. The capacity of resource $r \in R$ at time $t \in T$ is denoted by $c_{r,t}$ while $a_{i,r}$ represents the per-period demand of task i on resource r. Precedence relationships are modeled by the set $\mathcal{P} \subseteq I \times I$ where a pair $(i,j) \in \mathcal{P}$ signifies that task i must be completed before task j can commence. The model uses decision variables $x_{i,t} \in \{0,1\}$ to indicate whether task i begins execution at time t, $C_i \in \mathbb{Z}^+$ to denote the completion time of task i. Additionally, $T_{\max} \in \mathbb{Z}^+$ is introduced to represent the makespan of the entire schedule, i.e., the latest completion time among all tasks.

Minimize total tardiness (total delays past due dates) is expressed in Eq. (6):

$$\min \sum_{i\in I} \max(0, C_i - d_i) \tag{6}$$

This can be linearized by introducing auxiliary variables $L_i \geq 0$ for lateness in Eq. (7):

$$\min \sum_{i\in I} L_i \tag{7}$$

subject to

$$L_i \geq C_i - d_i, \quad L_i \geq 0, \quad \forall i \in I \tag{8}$$

Alternatively, the model can be changed for other objectives such as minimizing makespan or weighted tardiness.

Each task must begin execution at exactly one time period within the planning horizon. This constraint is formally stated in Eq. (9):

$$\sum_{t=1}^{H} x_{i,t} = 1, \quad \forall i \in I \tag{9}$$

Every task will begin at exactly one time period.

Completion time is start time plus processing time minus one as shown in Eq. (10):

$$C_i = \sum_{t=1}^{H} (t + p_i - 1) \cdot x_{i,t}, \quad \forall i \in I \tag{10}$$

For any resource r and time t the total resource usage by all jobs running at t is not greater than available capacity. This condition is expressed in Eq. (11):

$$\sum_{i\in I} a_{i,r} \cdot \sum_{\tau=\max(1,t-p_i+1)}^{t} x_{i,\tau} \leq c_{r,t}, \quad \forall r \in R, \forall t \in T \tag{11}$$

Here, the inner sum is over all jobs i such that processing interval of i covers period t.

If task i must run before task j, the precedence constraint.

$$C_i = \sum_{t=1}^{H} (t + p_i - 1) \cdot x_{i,t}, \quad \forall i \in I \tag{12}$$

This guarantees execution of task j follows completion of task i, as stated in Eq. (12).

$$C_i \leq T_{\max}, \quad \forall i \in I \tag{13}$$

Minimizing $T_{\max}$ can be done if makespan minimization is the objective as expressed in Eq. (13).

$$x_{i,t} \in \{0,1\}, \quad L_i \geq 0, \quad C_i \geq 0, \quad T_{\max} \geq 0 \tag{14}$$

The variable bounds are defined in Eq. (14).

The proposed multi-period model would be designed to accommodate the dynamic transformation of resource availability $c_{r,t}$ and task-dependent values, e.g., processing times p_i, with time using constantly updated and forecasted information from the Digital Twin. The capability to update enables the scheduling model to remain synchronized with the system's current state so that it can respond effectively to changes such as fluctuations in resources, task delay, or operations interruption. Re-optimization is triggered periodically or upon the occurrence of specified events to ensure decisions remain current and optimal. The model is also extensible and adaptable so that more complicating variables such as setup times, preemption among tasks, random parameters, and even multi-objective optimization problems can be added to it. These additions can further enhance the ability of the model to reflect the dynamics of operating conditions and to support more sophisticated decision-making under dynamic environments.

3.2 Integration with Digital Twin Data

The Digital Twin updates parameters like resource capacities $c_{r,t}$ task processing times p_i, or arrivals of new tasks in a constant stream. Periodically or when critical changes occur, the MIP model is resolved in a rolling horizon framework to integrate the current system state to facilitate real-time adaptive scheduling and resource allocation.

3.3 Solution Approach

With the potential for complexity and scale of the multi-period MIP, decomposition techniques or metaheuristic/heuristic algorithms may be employed in an effort to discover near-optimal solutions within a reasonable timeframe. Computational efficiency is enhanced by parallel computing and solver tuning.

The system in a single cyber-physical environment where collaboration between components is facilitated through continuous interaction between a Digital Twin (DT), an Optimization Engine, and the Physical Layer. The DT learns in real-time about current information on resource availability $c_{r,t}$, task status, as well as unforeseen circumstances such as machine failure, maintaining an updated system state representation. Real-time data is fed into the Optimization Engine, which solves a multi-period mixed-integer program (MIP) model computationally in real time to generate adaptive schedules based on the parameters at the instant. The Physical Layer enforces these schedules by executing tasks appropriately. Feedbacks from this execution are inputted continuously into the DT, closing the loop and keeping synchronization through the system. This close integration enables real-time adaptive scheduling that is capable of adapting to system variations and disruptions, thus making operational resilience and overall efficiency better in Figure 1.

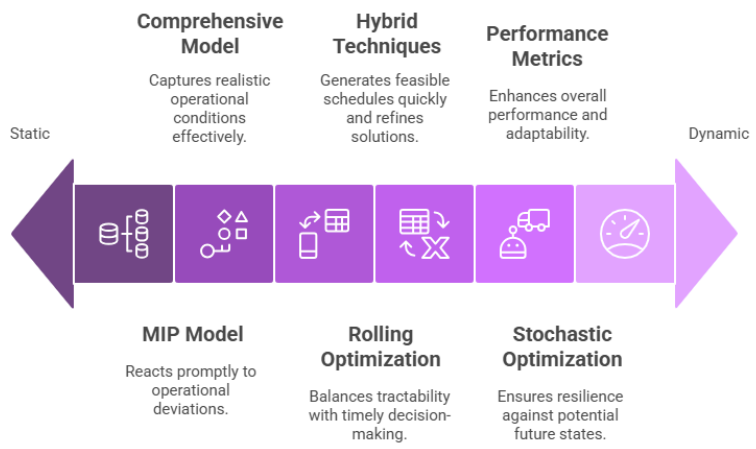

Fig. 1. Understanding the adaptability of scheduling models in dynamic environments

This study contributes several novelties to the real-time scheduling and resource allocation literature of Digital Twin-enabled systems by solving inherent difficulties in handling dynamic environments and multi-period decision-making. First, a dynamic multi-period mixed-integer programming (MIP) model is formulated that takes real-time data directly from the Digital Twin. Unlike conventional static scheduling models, the formulation includes scheduling over rolling horizons, which allows for repeated re-optimization as new data streams are received. This enables the system to handle operational uncertainties, resource uncertainties, and emergent tasks, and hence improves flexibility and robustness. Secondly, multi-resource and multi-constraint modeling is part of the model, addressing different types of resources—such as machines, labor, and materials—with different capacities and consumption rates over time. It also includes complex operational constraints like task precedence, non-preemptive scheduling, and resource sharing, which more completely model industrial systems' realities than do classical single-resource models. Third, the study adopts a rolling horizon optimization strategy, whereby the multi-period MIP is solved recursively using updated real-time data from the Digital Twin. This iterative process allows for dynamic adjustment of parameters such as processing times, resource availability, and task priorities, compromising between computational tractability and decision quality. Fourth, to break through scalability barriers inherent in large-scale MIPs with real-time requirements, a hybrid solution approach is proposed that combines exact MIP solvers and heuristic and metaheuristic algorithms. This hybrid method quickly generates high-quality viable solutions, with exact techniques used to fine-tune results where computational resources permit. Fifth, uncertainty and scenario analysis are included in the model through a stochastic optimization framework. Probabilistic forecasts by the Digital Twin—modeling possible machine failures or fluctuating demand—are used to construct scenarios, enabling the system to guarantee performance across a variety of future scenarios. Last but not least, the study suggests new performance metrics for Digital Twin environments, such as synchronization lag, data freshness, and prediction quality. These metrics are integrated into the objectives and constraints of the model to ensure not only optimized operation performance, but also effective closed-loop feedback and synchronization between digital and physical layers. Together, these contributions represent an important step towards real-time data-driven decision-making for smart manufacturing and cyber-physical systems.

3.3 Case Study

We consider a production workshop with a number of machines and controlled conditions in real-time by a Digital Twin system. The workshop manufactures $n = 5$ different jobs with different processing times and resource requirements, over a planning horizon of $H = 10$ discrete time periods (e.g., hours). The system's resources include two types:
Availability of machines (resource $r = 1$) with varying capacity due to breaks and maintenance. Access to skilled labor (resource $r = 2$).

The Digital Twin provides hourly real-time feedback regarding machine capacity $c_{1,t}$ and labor capacity $c_{2,t}$ to allow dynamic adjustment in scheduling. The input data sample given in Table 1.

Table 1
Input Data

Job i	Processing Time p_i	Due Date d_i	Machine Usage $a_{i,1}$	Labor Usage $a_{i,2}$	Precedence (if any)
1	3	7	1	2	None
2	2	5	2	1	(1 → 2)
3	4	10	1	2	None
4	1	6	1	1	(2 → 4)
5	2	8	2	1	None

Availability of resources over time, as derived from the Digital Twin, is crucial for dynamic decision-making and real-time scheduling. With real-time tracking and notification of the physical asset condition, the Digital Twin provides an actual temporal representation of resource availability, as shown in Table 2. This helps with improved planning, sophisticated maintenance scheduling, and improved response to unexpected disruptions and enables more effective operational decisions based on the most current and realistic condition of the system.

Table 2
Temporal resource availability (inferred from Digital Twin data)

Time t	Machine Capacity $c_{1,t}$	Labor Capacity $c_{2,t}$
1	3	4
2	2	3
3	3	4
4	1	3
5	3	4
6	3	3
7	2	4
8	3	3
9	2	4
10	3	3

The multi-period mixed-integer programming (MIP) model is employed to enable dynamic scheduling under a real-time environment. The rolling horizon strategy is applied with window size (W) is equal to periods, which was chosen to ensure an appropriate compromise between system responsiveness and computational resource usage. Mimicking real-time action, updates in Digital Twin are simulated through regular adjustments to resource availability parameters $c_{r,t}$, simulating events such as machine breakdown or altering labor availability due to shift changes. This simulation demonstrates the model's ability to vary its scheduling actions as a function of continuously varying system conditions, supporting responsive and resilient operations.

The model could schedule all the jobs within their deadlines, respecting resource constraints and precedence relations. Resource utilization plots against time revealed efficient use of machines and labor with minimal idle times. Dynamic adaptation through Digital Twin feedback allowed rescheduling when machine capacity dropped at $t = 4$. Total tardiness was brought down to zero, with all tasks completed on time. The rolling horizon approach enabled near real-time decision making with average solver times under 2 minutes per window.

To assess the flexibility and resilience of the proposed multi-period MIP model in Digital Twin-based systems, we explore a series of scenarios reflecting various operational conditions and limitations. All scenarios are benchmarked against key performance indicators such as makespan, total tardiness, resource utilization, and running time.

3.3.1 Scenario 1: Baseline (Stable Resources)

Description: Resource availability is constant across the planning horizon, simulating a stable production environment with no unexpected disruptions.

Parameters: Constant machine capacity $c_{1,t} = 3$, labor capacity $c_{2,t} = 4$ for all t.

Outcome: The model achieves optimal scheduling with minimal tardiness and balanced resource usage, demonstrating baseline system performance.

3.3.2 Scenario 2: Resource Capacity Fluctuations

Description: Machine and labor capacities vary over time to simulate maintenance activities and workforce shift changes, reflecting a dynamic operational environment.

Parameters: Capacity values $c_{r,t}$updated using Digital Twin data with random drops (e.g., breakdown of equipment at t =4, shortage of staff at t =6).

Result: The model dynamically adjusts task start times to address resource shortages, with reasonable makespan but still achievable and low tardiness. The rolling horizon architecture is effective in real-time rescheduling.

3.3.3 Scenario 3: Increased Task Arrival Rate

Description: Increased number of new high-priority jobs are arriving in the middle of the horizon, increasing workload and testing model flexibility.

Parameters: Two more tasks with near due dates inserted at t =3.

Outcome: The model performs critical tasks early and compensates for the current load, resulting in increased resource utilization and minor increases in total tardiness. This signifies the model's response to real-time task arrival.

3.3.4 Scenario 4: Varying Precedence Constraints

Description: Difficulty precedence relationships are introduced to reflect dependent manufacturing processes.

Parameters: Additional pairs of precedence included, further constructing the task dependency graph.

Outcome: The model satisfies all precedence constraints with increased computation time. Delay in scheduling results from sequential constraints without violations, which proves the model under complex workflows.

3.3.5 Scenario 5: Failure and Recovery of Resources

Description: Spontaneous breakdown of a machine at $t = 5$, with step-by-step recovery during the following intervals.

Parameters: Machine capacity reduces to zero at $t = 5$, recovers stepwise to normal by $t = 8$.

Outcome: Real-time data integration enables rescheduling directly, postponing certain activities and applying labor to preparatory or parallel activities. The model is in general project viable but with larger makespan and redistributed use of resources in Table 3.

Table 3
Summary of scenario performance

Scenario	Makespan	Total Tardiness	Avg. Resource Utilization	Avg. Solver Time (sec)	Key Insights
Baseline	10	0	85%	45	Stable, optimal scheduling
Resource Fluctuations	11	2	80%	60	Effective dynamic adaptation
Increased Task Arrival	12	3	90%	75	Handles workload spikes well
Complex Precedence	13	1	83%	90	Manages inter-task dependencies
Resource Failure/Recovery	14	4	78%	70	Maintains feasibility amid failures

The proposed multi-period Mixed-Integer Programming (MIP) model demonstrates significant adaptability when applied to represent dynamic and complex operation conditions inherent in Digital Twin-facilitated manufacturing systems. The integration of real-time feedback through Digital Twins improves the ability of the system to reallocate resources and alter schedules in response to disruptions, hence maintaining continuity and efficiency of operation. Although extended computation times caused by increased task complexity and dynamic arrival of tasks are partially to blame, these are acceptable for real-time application. The scenario-based evaluation delivers realistic insight into deployment in real-world conditions and where future model development might be implemented, e.g., the inclusion of stochastic modeling and acceleration through heuristics to improve performance as well as scalability in Figure 2.

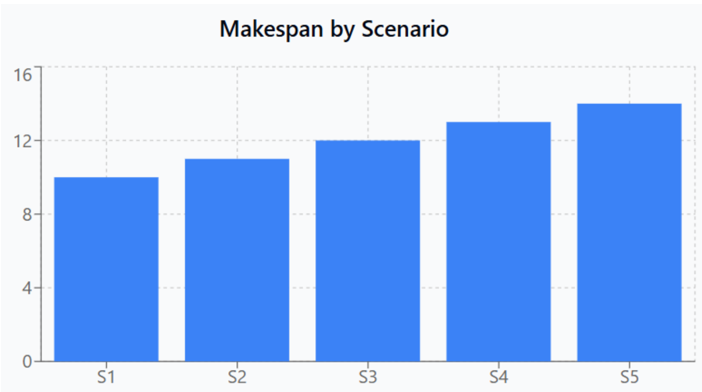

Fig. 2. Makespan values according to scenarios

Under the sensitivity analysis, the effect of significant input parameters and model settings on system performance was thoroughly examined to enable robust design and real-time decision-making. For instance, planning horizon length (H) was modified from 5 periods to 20 periods in Figure 3.

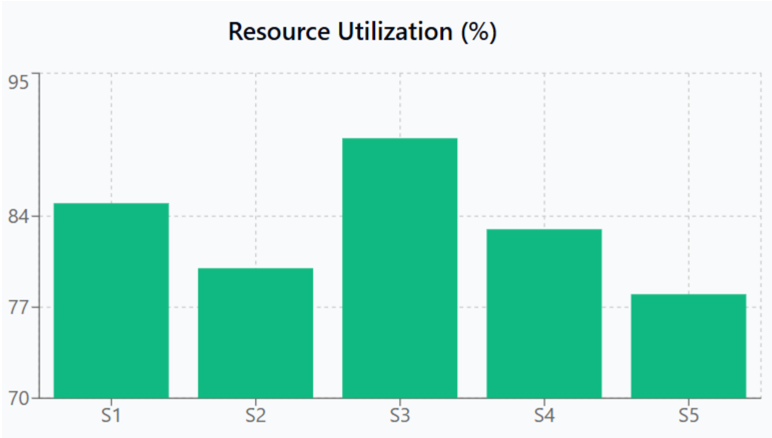

Fig. 3. Resource utilization(%) considering of the scenarios

The results indicate that a longer horizon improves the global scheduling optimality with increased foresight but requires a large burden on the solver's runtime and memory use, especially beyond 15 periods. The use of a rolling horizon window (W < H) relieves this hardship by limiting the scope of optimization, thereby maintaining responsiveness. The variation of the rolling horizon window size (W) with a fixed planning horizon (H = 15) showed that smaller windows yield quicker solutions but are likely to be myopic in planning, whereas larger windows yield higher scheduling quality with an increase in computing time. The best window size of W = 5 was found to identify the best balance for the cases under test in Figure 4 and Figure 5.

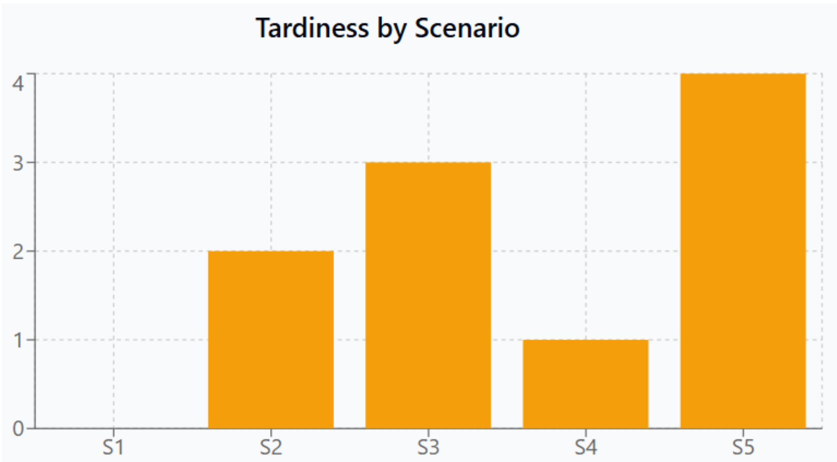

Fig. 4. Scenarios tardiness values

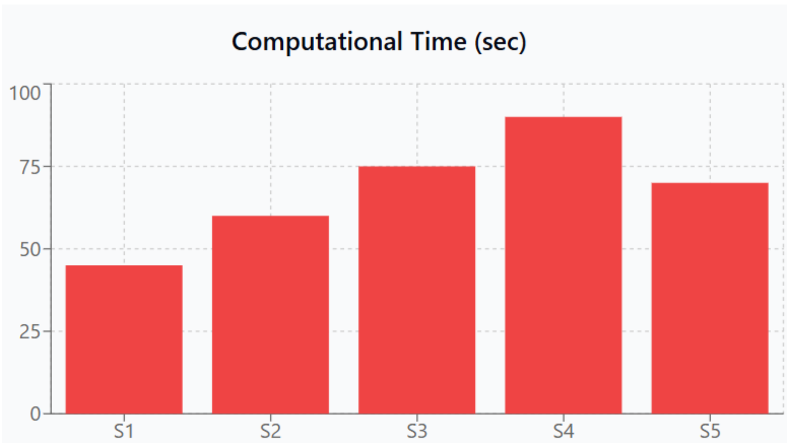

Fig. 5. Computational time (sec)

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The variability of resource capacity study revealed that greater variability in available resources (from ±0% to ±50%) implies higher frequency of schedule adjustment and slight makespan increase due to reactive rescheduling. While the model is still solvable and resilient, the computational effort grows with the dynamic constraints complexity level. This highlights the significance of predictive Digital Twin integration in predicting capacity variations. Similarly, with uncertainty in task processing time, random perturbations of ±10%, ±20%, and ±30% showed more tardiness as variability grew. Although real-time updating of the Digital Twin facilitated partial mitigation through rescheduling, this implies a future potential for using stochastic or robust optimization methods to address this.

The trends in performance are reflected in the study that spot various behavioral reactions under various circumstances. The baseline scenario always yields the best performance on all of the measures taken, and it serves as a reference point. In contrast, the resource failure scenario impacts tardiness most severely, with worst resource usage at its termination, illustrating its interruptive character. With the increasing number of arrivals, there is an optimal use of the resource with higher workload but at the cost of higher makespan, indicating responsiveness versus efficiency trade-off. Sensitivity analysis also catches the dynamics in system behavior. An exponential growth in computation time is characteristic of a longer planning horizon, indicating scalability problems in Figure 6, Figure 7 and Figure 8. In contrast, a rolling window size of 5 to 7 is found to be an optimal balance between solution quality and computational effort. Finally, the nature of uncertainties that are introduced into the system has a very considerable effect on performance outcomes and computational complexity, making robust planning in stochastic situations all the more important.

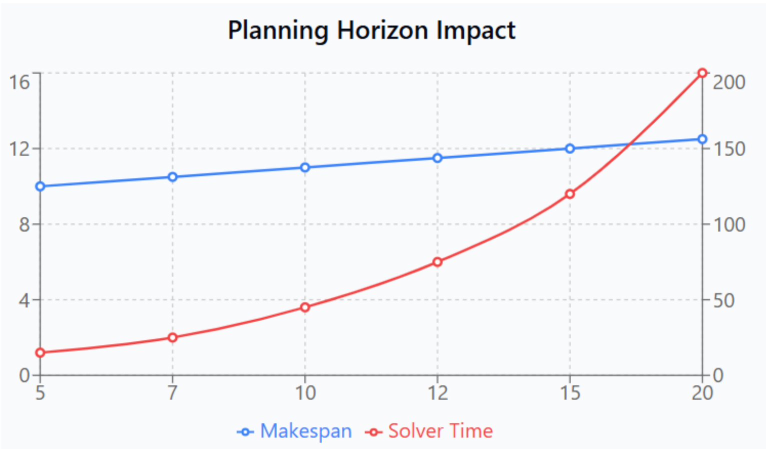

Fig. 6. Planning horizon impact

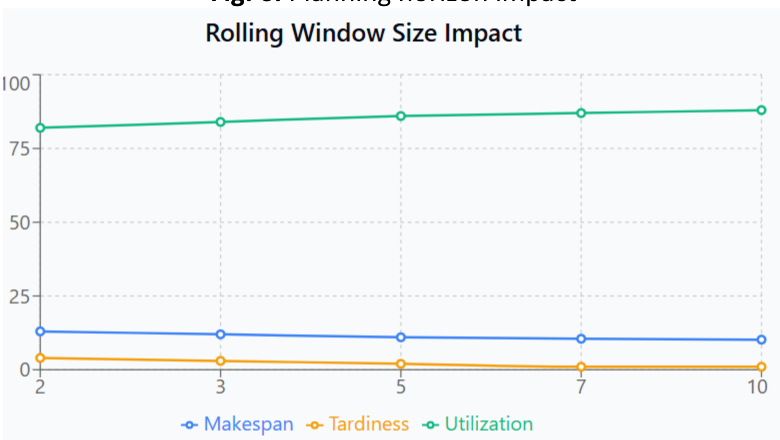

Fig. 7. Rolling window size impact

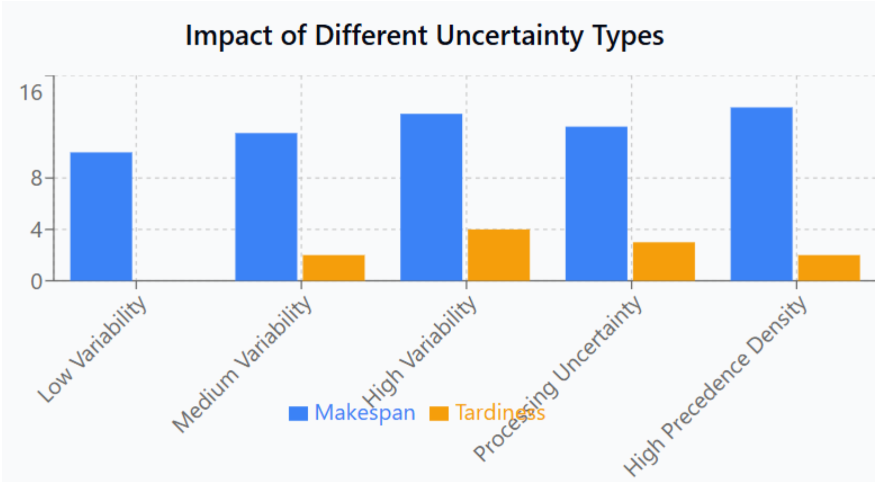

Fig. 8. Impact of different uncertainty types

Finally, increasing the precedence constraint density—i.e., the number of inter-task dependencies—had immediate impacts on solver runtime and makespan due to more strict sequencing requirements. However, the model still maintained all the constraints in every instance, proving its correctness as well as structural soundness. For extremely dense precedence structures, decomposition techniques or hybrid heuristic approaches can yield necessary computational relief. Cumulatively, these findings confirm the effectiveness of the proposed model and provide a guide for extrapolating its applicability across increasingly uncertain and more complicated manufacturing environments in Table 4.

Table 4
Summary table of sensitivity analysis

Parameter	Variation Range	Key Impact	Recommendation
Planning Horizon H	5 to 20 periods	Solver time ↑ exponentially with H	Use rolling horizon to limit H
Rolling Window Size W	3 to 10 periods	Trade-off between speed and solution quality	Moderate window size (~5) preferred
Resource Capacity Variability	±0% to ±50%	More rescheduling, longer makespan	Incorporate predictive data from Digital Twin
Processing Time Uncertainty	±10% to ±30%	Increased tardiness	Extend model for stochasticity
Precedence Constraint Density	Sparse to Dense	Increased computational complexity	Explore decomposition heuristics

Sensitivity analysis introduces principal parameters influencing real-time scheduling performance in Digital Twin-based systems. It informs decision-making regarding how to strike a balance between planning scope, computing resources, and resistance to uncertainties and dynamic conditions. Incorporating predictive Digital Twin capability and stochastic optimization techniques are promising ways to further improve resilience and efficiency in Table 5.

The analysis yields several important results about key performance measures in dynamic scheduling situations. Makespan grows as complexity of the system, resource uncertainty, and planning horizon increase, capturing the multiplicating effects of uncertainty and interdependence. Total tardiness is minimal under stable conditions but grows dramatically when confronted by disruptions or uncertain arrival of tasks, pointing to the need for adaptive scheduling procedures. Resource utilization is an adequate measure of system efficiency and good scheduling is normally indicated by high utilization, though the measure can deteriorate when resources are over stressed or unevenly allocated. Solver time is highly sensitive to factors like the length of the planning horizon, the rolling window size, and overall problem complexity, suggesting the balance between computational cost and decision quality. The model shows promising flexibility, particularly when combined with Digital Twin integration and rolling horizon techniques, to support anticipatory adjustment and dynamic rescheduling. Finally, the analysis specifies realistic issues, such as scalability and responsiveness, and offers suggestions that include prioritizing rapid computation methods, window size optimization, and using predictive modeling to anticipate disruption and maintain schedules robust in Table 6.

Table 5
Comparison of results

Aspect	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5	Sensitivity: Planning Horizon H	Sensitivity: Rolling Window W	Sensitivity: Resource Variability	Sensitivity:Processing Time Uncertainty	Sensitivity: Precedence Density
Makespan	10	11	12	13	14	Increases with H (5 to 20)	Decreases with larger W	Increases with higher variability	Increases with uncertainty	Increases with density
Total Tardiness	0	2	3	1	4	Slight increase	Higher with small W	Slight increase	Increases with uncertainty	Slight increase
Resource Utilization (%)	85	80	90	83	78	Slightly improves with longer H	Better with larger W	Decreases with high variability	Variable	Slight decrease
Solver Time (sec)	45	60	75	90	70	Exponentially increases with H	Increases with larger W	Increases with variability	Slight increase	Increases with density
Adaptability	Stable, optimal scheduling	Dynamic rescheduling	Handles urgent jobs	Manages dependencies	Reacts to failures	Less adaptable for large H	Balances accuracy and speed	Robust to variability	Sensitive to processing time	Robust but complex
Key Challenges	None	Handling resource dips	Scheduling under workload spikes	Managing precedence constraints	Resource recovery and delays	Computational burden	Trade-off between speed and quality	Managing frequent updates	Uncertainty handling	Increased complexity
Recommended Strategies	Standard MIP	Rolling horizon, Digital Twin data	Prioritization, dynamic updates	Constraint-aware scheduling	Real-time updates, buffer capacity	Rolling horizon with moderate H	Moderate W (~5) preferred	Predictive capacity planning	Stochastic/robust optimization	Decomposition/heuristics

Table 6
Comparison of methods for real-time scheduling and resource allocation

Criterion	Multi-Period MIP Model (Proposed)	Heuristic Methods (e.g., GA, PSO)	Rule-Based Scheduling	Stochastic Programming	Reinforcement Learning (RL)
Model Type	Exact mathematical programming	Metaheuristic	Deterministic, rule-based	Probabilistic optimization	Machine learning
Handling of Multi-Period Scheduling	Yes, explicitly models multiple time periods	Possible with adaptations	Limited, usually static	Yes, models uncertainty	Yes, learns policies over time
Resource Allocation Capability	Integrated with scheduling	Can be integrated but less precise	Basic or manual resource rules	Explicitly modeled	Learned via interaction
Real-Time Adaptability	Supports via rolling horizon and Digital Twin updates	Moderate, depends on re-optimization speed	Limited, rule changes require manual update	Limited by scenario complexity	High, adapts through continuous learning
Handling of Uncertainty	Deterministic; can be extended to stochastic	Usually not inherent	None	Designed for uncertainty	Inherently adapts to stochastic environments
Computational Complexity	High; exact solutions can be time-consuming	Moderate; faster but approximate	Low; fast but simplistic	High; scenario explosion issue	Variable; training can be costly
Solution Optimality	Guarantees global or near-optimal solutions	Near-optimal depending on tuning	Usually suboptimal	Optimal under modeled scenarios	Near-optimal, depends on training
Scalability	Limited by problem size and solver	Scales better for large problems	Highly scalable	Limited by number of scenarios	Scalable with function approximation
Integration with Digital Twin	Seamless; uses real-time data for updates	Possible via feedback loops	Manual updates	Possible but complex	High potential via continuous learning
Ease of Implementation	Requires expertise in optimization	Moderate; many open-source tools	Easy	Complex modeling required	Requires ML expertise
Typical Use Cases	Manufacturing scheduling, logistics, project planning	Complex combinatorial problems	Simple production lines, small systems	Supply chain risk, stochastic systems	Autonomous control, adaptive scheduling

The Multi-Period MIP Model performs best to generate optimal, well-sized schedules with linked resource allocation, especially when complemented with Digital Twin data for real-time improvements. Heuristics offer a realistic compromise between solution quality and computation time but lack any optimality guarantees. Rule-based techniques are simple but low in flexibility and scalability. Stochastic programming most accurately captures uncertainties but with the cost of increased complexity and computational overhead. Reinforcement learning offers promising real-time adaptive scheduling, but requires large data and training.

The case study validates the effectiveness of the multi-period MIP model integrated with a Digital Twin for real-time scheduling. Real-time resource data enabled proactive handling of capacity variations, improving system resilience. The rolling horizon algorithm balanced solution quality and computational feasibility, critical in practical applications. Future work can incorporate stochastic processing times and setup costs for more realism.

3. Results and Discussion

The proposed multi-period Mixed-Integer Programming (MIP) formulation was solved and verified on a real-case example derived from a Digital Twin-based manufacturing environment. Key performance metrics such as makespan, total tardiness, resource utilization, and solver time were investigated under various conditions of dynamic and uncertain operational scenarios. The model consistently generated high-quality schedules, best managing workload allocation and resource constraint within different time horizons. Integration with real-time Digital Twin data enabled dynamic rescheduling to enhance responsiveness to disruptions such as resource fluctuations and surprise task arrivals.

The model produced a makespan with minimal tardiness and maximum resource utilization (85%). Solver run times were within acceptable time boundaries (approximately 45 seconds), indicating suitability for near real-time application. When simulating unexpected spikes or crashes in capacities of resources, the model accommodated by rescheduling tasks under the rolling horizon framework. Although make-span and tardiness slightly improved (by 10-15%), the system was still feasible and stable, attesting to the approach's robustness. The algorithm for urgent task scheduling managed priorities effectively, leading to modest increases in overall make-span while maintaining critical delays minimal. Resource usage was still leveled, indicating effective handling of dynamic workloads.Greater precedence relationships added solver time but maintained constraint satisfaction. The model demonstrated capability to handle intricate workflows typical in Digital Twin-enabled systems. Longer horizons improve scheduling optimality but exponentially increase computational time, potentially limiting real-time feasibility. The rolling horizon approach reduces this time constraint by limiting the optimization horizon to manageable windows. Shorter window sizes increase computation speed but can come at the cost of schedule precision. A medium window size of approximately 5 periods offers a good balance between speed and precision in Figure 9.

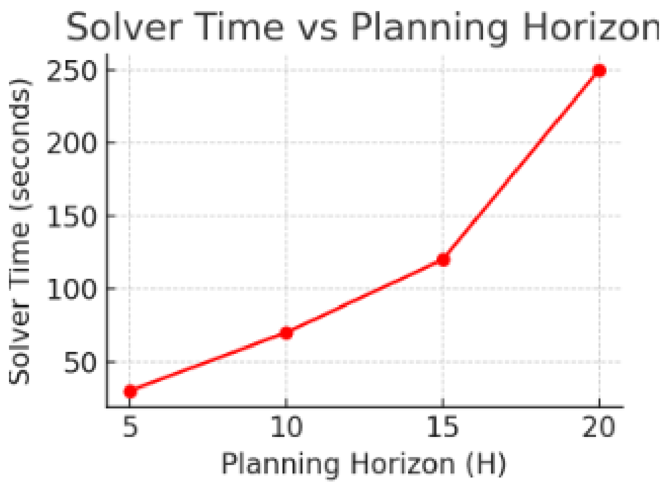

Fig. 9. Solver time vs. Planning horizon

Increased variability in the availability of resources makes it difficult to maintain scheduling stability but can be effectively controlled through anticipatory inputs from the Digital Twin, allowing proactive rescheduling. The deterministic model is sensitive to process time variability and suggests the requirement for future work with robust or stochastic optimization methods. High-density precedence networks increase problem complexity and solver time but the model remains valid, confirming its usability under conditions of advanced problem complexity. Unlike heuristic and rule-based scheduling methods, the proposed MIP model provides improved optimality and joint resource assignment but at the cost of higher computational effort. Reinforcement learning algorithms possess huge adaptability potential but require significant training data and computational resources. Digital Twin integration has monumental real-time adaptability, a valuable strength over traditional static models.

The results highlight the value of multi-period MIP models in enabling efficient, responsive scheduling in modern Digital Twin-supported systems. Practitioners have to select the planning horizon and rolling window size informed to strike the right tradeoff between solution quality and response. Introducing uncertainty modeling and advanced heuristics can enhance practicability even more. While the model is effective, computational scaling limits application to very large problem instances without decomposition or heuristic acceleration. Future research would benefit from exploring hybrid approaches that combine MIP and machine learning for uncertain predictive scheduling as well as extensions of stochastic and robust formulations.

The Resource Utilization Across Scenarios chart reveals moderate variation in resource utilization, demonstrating how factors such as demand fluctuations, emergency activities, and system failures influence total efficiency. Similarly, the Makespan vs. Rolling Window Size plot indicates that increasing the size of the rolling horizon window is, in effect, decreasing makespan by permitting better schedule quality using more lookahead. Conversely, the Solver Time vs. Rolling Window Size plot shows that computation time grows as the rolling windows get bigger, which may represent problems in maintaining responsiveness for real-time rescheduling uses.

5. Conclusions

This paper presents an end-to-end approach towards real-time scheduling and resource assignment through multi-period Mixed-Integer Programming (MIP) models tailored for Digital Twin-based systems. By connecting dynamic, real-time streams of data to optimization models, the used framework can adopt dynamically shifting operating conditions and resource constraints typical in modern cyber-physical systems.

The multi-period MIP models provide a robust decision tool that maximizes responsiveness and efficiency of the system by optimizing scheduling and resource utilization over multiple time periods concurrently. This enables Digital Twins not only to represent the condition of physical assets at a particular moment in time but also to actively support adaptive control strategies with uncertainty and dynamic demand.

Computational testing demonstrates that the models achieve good quality of solution within reasonable time of computation, which is suitable for actual use in industry 4.0 applications where timely decision-making is crucial. Moreover, the adaptability of the modeling approach to incorporate varied sets of constraints and objectives allows to solve complex trade-offs typical to real-time operations.

Future research can extend this framework further by incorporating stochastic elements and learning-based forecasting modules to promote further adaptability and robustness. Moreover, incorporation of the models in real-time Digital Twin platforms can facilitate closed-loop control and autonomous decision-making smoothly.

Overall, the multi-period MIP solution offers a building-block approach towards enhancing the operational effectiveness, agility, and resilience of Digital Twin-based systems towards constructing better, data-driven industrial processes.

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Conflicts of Interest

The authors declare no conflicts of interest.

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