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Multi-Criteria Decision-Making Method Based on Probabilistic Uncertain Linguistic T-Spherical Fuzzy ARAS

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ABSTRACT

Based on the need to address the uncertainty and fuzziness of evaluation information in multi-criteria decision-making while reflecting probability distributions, a novel Probabilistic Uncertain Linguistic T-Spherical Fuzzy Set (PULTSFS) is proposed by integrating the probabilistic expression advantages of PULTS with the three-dimensional evaluation characteristics of TSFS. Firstly, the related concepts and fundamental operational rules of PULTSFS are defined, including the score function, accuracy function, Hamming distance measure, and the Probabilistic Uncertain Linguistic T-Spherical Fuzzy Weighted Averaging (PULTSFWA) operator, with properties such as monotonicity, idempotency, and boundedness being analyzed. Subsequently, the ARAS method is extended to establish a multi-criteria decision-making model based on PULTSF-ARAS. In this model, the criterion weights are determined by integrating subjective weights and objective distance-based weights, and the relative closeness of alternatives is calculated using the positive ideal solution to rank the alternatives. Finally, a case study on green supplier selection for new energy vehicle enterprises is conducted to demonstrate the validity and feasibility of the proposed method.

1. Introduction

Multi-criteria decision-making is a vital branch of modern decision science, with its research increasingly focusing on characterizing fuzzy and uncertain information. To overcome the limitation of classical fuzzy sets containing only membership degrees [1], Atanassov [2] proposed Intuitionistic Fuzzy Sets (IFS), introducing both membership and non-membership degrees. Yager [3] further developed Pythagorean Fuzzy Sets (PyFS), relaxing the constraints to the sum of squares. Subsequently, he proposed q-order rectified fuzzy sets (q-ROFS) [4], extending expressive power through qth-power operations. Addressing “neutral” information, Cuong *et al.,* [5] introduced pictorial fuzzy sets (PFS), which Mahmood *et al.,* [6] extended to spherical fuzzy sets (SFS) and T-spherical fuzzy sets (TSFS). The latter satisfies the constraint that the qth-power sum of

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membership, neutrality, and non-membership does not exceed 1, making it an effective tool for expressing complex fuzzy information [7,8]. In linguistic expression, Zadeh [9] introduced linguistic term sets (LTS); Rodriguez *et al.,* [10] introduced hesitant fuzzy linguistic term sets (HFLTS); Pang *et al.,* [11] proposed probabilistic linguistic term sets (PLTS), assigning probabilities to terms to reduce information loss; Lin *et al.,* [12] further proposed the probabilistic uncertain linguistic term set (PULTS), which combines fuzziness, hesitancy, and uncertainty. It has been applied in scenarios such as technology route selection [13] and supplier evaluation [14], with scholars conducting generalization studies like PULIFS [15] and PULq-ROFS [16]. Although PULq-ROFS possesses broader information expressiveness than PULIFS and PULTS, it characterizes evaluation information solely through membership and non-membership degrees, failing to capture neutrality levels within evaluation data. Consequently, addressing complex evaluation and decision-making problems urgently requires a new tool capable of expressing more comprehensive and extensive evaluation information.

The ARAS method is a multi-attribute decision-making approach. It ranks alternatives by comparing each option against a "perfect solution," featuring a simple and intuitive principle with a clear computational process. By introducing an explicit perfect solution as a benchmark, it ensures excellent comparability of evaluation results [17]. In the relevant literature, ARAS technology has been applied in explicit numerical environments [18], fuzzy environments [19], and its extended forms (e.g., interval-valued fuzzy sets, type-2 fuzzy sets, etc.) [20-22]. Büyüközkan and Güler [21] further extended the ARAS method to hesitant fuzzy environments. Building upon this, Iordache *et al.,* [22] implemented the ARAS method on interval-type hesitant fuzzy sets. Collectively, these studies demonstrate that the ARAS method provides efficient computational and evaluation solutions. However, while ARAS offers intuitive comparability, its ratio serves as a relative scale that obscures the absolute magnitude and true differences between attributes, potentially compromising the integrity of decision information. Therefore, necessary improvements to the original method are crucial.

Current multi-criteria decision research must address complex information such as linguistic hesitation, cognitive fuzziness, and neutral attitudes coexisting in evaluators' assessments. Existing decision models lack a comprehensive framework to represent such multidimensional uncertainty. The integration of Probabilistic Uncertain Language Sets (PULS) with T-Spherical Fuzzy Sets (TSFS) effectively addresses this limitation. PULS precisely captures the probabilistic distribution characteristics in decision-makers' linguistic evaluations, while TSFS comprehensively expresses their support, neutrality, and opposition attitudes through three dimensions: membership, neutrality, and non-membership. Simultaneously, extending the ARAS method with a PULTSF-based Hamming distance comparison mechanism provides an intuitive, reliable, and precise computational framework for complex information environments. Thus, constructing an ARAS-extended decision model based on PULTSF not only broadens the integration scope between fuzzy set theory and decision methods but also offers new methodological support for handling complex decision problems under high uncertainty. This approach serves as a novel tool capable of expressing more comprehensive and extensive evaluation information.

2. Preliminaries

2.1 PULTS and TSFS

To precisely express decision-makers' concerns, Lin *et al.,* [23] proposed the concept of Probabilistic Uncertain Linguistic Term Sets (PULTS) based on Uncertain Linguistic Variables (ULVs) [24] and Probabilistic Linguistic Term Sets (PLTS) [11].

Definition 1 [23]. A PULTS is defined as follows:

$$UL(p) = \left\{\langle\left[s_\alpha^{(t)}, s_\beta^{(t)}\right], p^{(t)}\rangle \,\middle|\, p^{(t)} \geq 0, t = 1,2,\ldots,\#T, \textstyle\sum_{t=1}^{\#T} p^{(t)} \leq 1\right\} \quad (1)$$

where $\langle\left[s_\alpha^{(t)}, s_\beta^{(t)}\right], p^{(t)}\rangle$ denote the t^{th} PUL element in *UL*(*p*), with $s_\alpha^{(t)}, s_\beta^{(t)}$ representing the lower and upper linguistic terms respectively, and $p^{(t)}$ being the corresponding probability.

Compared to traditional fuzzy sets and their variants, TSFS offers experts a broader expressive space and greater flexibility in three dimensions. Mahmood *et al.,* [6] provide the following definition:

Definition 2 [6, 25]. Let X={x1,x2,...,xn} denote a domain. A TSFS A over X is an object conforming to the following form:

$$A = \left\{\langle x_j, \left(\mu_A(x_j), \eta_A(x_j), \nu_A(x_j)\right)\rangle \middle| x_j \in X\right\} \quad (2)$$

where $\mu_A(x_j), \eta_A(x_j), \nu_A(x_j): X \to [0,1]$ respectively denote the membership degree, neutrality degree, and non-membership degree of x_j for *A*, satisfying the condition $0 \leq \left(\mu_A(x_j)\right)^q + \left(\eta_A(x_j)\right)^q + \left(\nu_A(x_j)\right)^q \leq 1 (q \geq 1)$. For computational convenience, a triplet$a_j = \left(\mu_j, \eta_j, \nu_j\right)$ is termed a TSFN. The parameter *q* can be appropriately set: a larger *q* value indicates weaker commitment, while a smaller *q* value implies less hesitation and reduced uncertainty.

2.2 PULTSFS

Building upon the strengths of PULTS and TSFS, this paper defines a novel probabilistic fuzzy set, PULTSFS, which not only permits experts to express evaluation information using multiple linguistic terms across three dimensions but also incorporates the probability associated with each linguistic term. The definition of PULTSFS is as follows:

Definition 3. Let X be a field and $S_{[0,k-1]}$ be an LTS. Then define a PULTSFS $\tilde{A}(p)$ over *X* as follows:

$$\tilde{A}(p) = \left\{\langle x_\varsigma, \varphi_\varsigma(\hat{p})(x_\varsigma), \phi_\varsigma(\tilde{p})(x_\varsigma), \psi_\varsigma(\bar{p})(x_\varsigma)\rangle \middle| x_\varsigma \in X\right\} \quad (3)$$

where $\varphi_\varsigma(\hat{p})(x_\varsigma) = \left\{\left[s_{\mu^L_{\varsigma(t)}}, s_{\mu^U_{\varsigma(t)}}\right](\hat{p}_{(t)}) \middle| s_{\mu^L_{\varsigma(t)}}, s_{\mu^U_{\varsigma(t)}} \in S_{[0,k-1]}, \hat{p}_{(t)} \geq 0, \sum_{t=1}^{\#T} \hat{p}_{(t)} \leq 1\right\}$ denotes the membership degree of $x_\varsigma \in X$; $\phi_\varsigma(\tilde{p})(x_\varsigma) = \left\{\left[s_{\eta^L_{\varsigma(r)}}, s_{\eta^U_{\varsigma(r)}}\right](\tilde{p}_{(r)}) \middle| s_{\eta^L_{\varsigma(r)}}, s_{\eta^U_{\varsigma(r)}} \in S_{[0,k-1]}, \tilde{p}_{(r)} \geq 0, \sum_{r=1}^{\#R} \tilde{p}_{(t)} \leq 1\right\}$ denotes the neutrality degree of $x_\varsigma \in X$; $\psi_\varsigma(\bar{p})(x_\varsigma) = \left\{\left[s_{\nu^L_{\varsigma(w)}}, s_{\nu^U_{\varsigma(w)}}\right](\bar{p}_{(w)}) \middle| s_{\nu^L_{\varsigma(w)}}, s_{\nu^U_{\varsigma(w)}} \in S_{[0,k-1]}, \bar{p}_{(w)} \geq 0, \sum_{w=1}^{\#W} \bar{p}_{(w)} \leq 1\right\}$ denotes the non-membership degree of $x_\varsigma \in X$, with corresponding probabilities $\hat{p}_{(t)}$, $\tilde{p}_{(r)}$ and $\bar{p}_{(w)}$; for $x_\varsigma \in X$, they satisfy the condition $0 \leq \left(max_{t=1}^{\#T} \mu^U_{\varsigma(t)}\right)^q + \left(max_{r=1}^{\#R} \eta^U_{\varsigma(t)}\right)^q + \left(max_{w=1}^{\#W} \nu^U_{\varsigma(w)}\right)^q \leq k^q$ (q≥1).

If *X* = {*x*}, then PULTSFS $\tilde{A}(p)$ degenerates into a PULTSFN $\tilde{a}(p)$, that is,

$$\tilde{a}(p) = \langle\left\{\left[s_{\mu^L_{(t)}}, s_{\mu^U_{(t)}}\right] \middle| \hat{p}_{(t)}\right\}, \left\{\left[s_{\eta^L_{(r)}}, s_{\eta^U_{(r)}}\right] \middle| \tilde{p}_{(r)}\right\}, \left\{\left[s_{\nu^L_{(w)}}, s_{\nu^U_{(w)}}\right] \middle| \bar{p}_{(w)}\right\}\rangle \quad (4)$$

where $s_{\mu^L_{(t)}}, s_{\mu^U_{(t)}}, s_{\eta^L_{(r)}}, s_{\eta^U_{(r)}}, s_{\nu^L_{(w)}}, s_{\nu^U_{(w)}} \in S_{[0,k-1]}, \sum_{t=1}^{\#T} \hat{p}_{(t)} \leq 1, \sum_{r=1}^{\#R} \tilde{p}_{(t)} \leq 1, \sum_{w=1}^{\#W} \bar{p}_{(w)} \leq 1$.

Note: For different values of parameter *q*, the PULTSFS $\tilde{A}(p)$ degenerates into distinct specific forms, as follows:

(1) When $\phi_\varsigma(\tilde{p})(x_\varsigma) = 0$, $\tilde{A}(p)$ degenerates into a probabilistic uncertain linguistic *q*-rung orthopair fuzzy set (PULqROFS)[16];

(2) When *q*=1 and $\phi_\varsigma(\tilde{p})(x_\varsigma) = 0$, $\tilde{A}(p)$ degenerates into a probabilistic uncertain linguistic intuitionistic fuzzy set (PULIFS) [17];

(3) When s$s_{\mu^L_{(t)}} = s_{\mu^U_{(t)}}$,$s_{\eta^L_{(r)}} = s_{\eta^U_{(r)}}$ and $s_{\nu^L_{(w)}} = s_{\nu^U_{(w)}}$, $\tilde{A}(p)$ degenerates into a probabilistic linguistic T-spherical fuzzy set (PLTSFS) [26];

(4) When $\phi_\varsigma(\tilde{p})(x_\varsigma) = 0$,$s_{\mu^L_{(t)}} = s_{\mu^U_{(t)}}$and $s_{\nu^L_{(w)}} = s_{\nu^U_{(w)}}$, $\tilde{A}(p)$ degenerates into a probabilistic linguistic q-rung orthopair fuzzy set (PLq-ROFS) [27];

Evidently, PULTSFS exhibits strong generality and can degenerate into specific forms under certain special conditions.

Definition 4. Let any two PULTSFNs, $\tilde{a}_1(p) = \langle\left\{\left[s_{\mu^L_{1(t)}}, s_{\mu^U_{1(t)}}\right] \middle| \hat{p}_{1(t)}\right\}, \left\{\left[s_{\eta^L_{1(r)}}, s_{\eta^U_{1(r)}}\right] \middle| \tilde{p}_{1(r)}\right\}, \left\{\left[s_{\nu^L_{1(w)}}, s_{\nu^U_{1(w)}}\right] \middle| \bar{p}_{1(w)}\right\}\rangle$ and $\tilde{a}_2(p) = \langle\left\{\left[s_{\mu^L_{2(t)}}, s_{\mu^U_{2(t)}}\right] \middle| \hat{p}_{2(t)}\right\}, \left\{\left[s_{\eta^L_{2(r)}}, s_{\eta^U_{2(r)}}\right] \middle| \tilde{p}_{2(r)}\right\}, \left\{\left[s_{\nu^L_{2(w)}}, s_{\nu^U_{2(w)}}\right] \middle| \bar{p}_{2(w)}\right\}\rangle$, be given. To facilitate computation, they need to be normalized as follows:

(1) (Probability Normalization) If $0 < \sum_{t=1}^{\#T} \hat{p}_{j(t)} < 1$ taking membership probability as an example, *j*=1,2), then $\tilde{a}_j(p)$ is normalized to $\tilde{a}_j(p^n)$, where the probability $\hat{p}^n_{j(t)} = \hat{p}_{j(t)} / \sum_{t=1}^{\#T} \hat{p}_{j(t)}$.

(2) (Structural Normalization) If $\#T_1 \neq \#T_2$ (taking membership degrees as an example), it is necessary to add some linguistic terms to the one with fewer elements, with their probabilities set to 0.

Definition 5. Let $S_{[0,k-1]}$ be a linguistic term set (LTS), and $\tilde{a}(p)$ be a probabilistic uncertain linguistic term set fuzzy number (PULTSFN), where $s_{\mu^L_{(t)}}, s_{\mu^U_{(t)}}, s_{\eta^L_{(r)}}, s_{\eta^U_{(r)}}, s_{\nu^L_{(w)}}, s_{\nu^U_{(w)}} \in S_{[0,k-1]}$, (*t*=1,2,...,#*T*; *r*=1,2,...,#*R*; *w*=1,2,...,#*W*), The score function of $\tilde{a}(p)$ is defined as:

$$Sc(\tilde{a}(p)) = s_{\frac{1}{2}\left(1+\left(\frac{\sum_{t=1}^{\#T}\left(\frac{1}{2}\left(\mu^L_{(t)}+\mu^U_{(t)}\right)\hat{p}_{(t)}\right)}{k\sum_{t=1}^{\#T}\hat{p}_{(t)}}\right)^q-\left(\frac{\sum_{r=1}^{\#R}\left(\frac{1}{2}\left(\eta^L_{(t)}+\eta^U_{(t)}\right)\tilde{p}_{(r)}\right)}{k\sum_{r=1}^{\#R}\tilde{p}_{(r)}}\right)^q-\left(\frac{\sum_{w=1}^{\#W}\left(\frac{1}{2}\left(\nu^L_{(t)}+\nu^U_{(t)}\right)\bar{p}_{(w)}\right)}{k\sum_{w=1}^{\#W}\bar{p}_{(w)}}\right)^q\right)} \tag{6}$$

The accuracy function of $\tilde{a}(p)$ is defined as:

$$Ac(\tilde{a}(p)) = s_{\left(\frac{\sum_{t=1}^{\#T}\left(\frac{1}{2}\left(\mu^L_{(t)}+\mu^U_{(t)}\right)\hat{p}_{(t)}\right)}{k\sum_{t=1}^{\#T}\hat{p}_{(t)}}\right)^q+\left(\frac{\sum_{r=1}^{\#R}\left(\frac{1}{2}\left(\eta^L_{(t)}+\eta^U_{(t)}\right)\tilde{p}_{(r)}\right)}{k\sum_{r=1}^{\#R}\tilde{p}_{(r)}}\right)^q+\left(\frac{\sum_{w=1}^{\#W}\left(\frac{1}{2}\left(\nu^L_{(t)}+\nu^U_{(t)}\right)\bar{p}_{(w)}\right)}{k\sum_{w=1}^{\#W}\bar{p}_{(w)}}\right)^q} \tag{7}$$

Definition 6. Assuming there are two arbitrary PULTSFNs, $\tilde{a}_1(p)$ and $\tilde{a}_2(p)$, the rules for comparing them are as follows:

(1) If $\mathrm{Sc}(\tilde{a}_1(p)) \succ \mathrm{Sc}(\tilde{a}_2(p))$, then $\tilde{a}_1(p) \succ \tilde{a}_2(p)$;

(2) If $\mathrm{Sc}(\tilde{a}_1(p)) \prec \mathrm{Sc}(\tilde{a}_2(p))$, then $\tilde{a}_1(p) \prec \tilde{a}_2(p)$;

(3) If $\mathrm{Sc}(\tilde{a}_1(p)) = \mathrm{Sc}(\tilde{a}_2(p))$, then

i) If $\mathrm{Ac}(\tilde{a}_1(p)) \succ \mathrm{Ac}(\tilde{a}_2(p))$, then $\tilde{a}_1(p) \succ \tilde{a}_2(p)$;

ii) If $\mathrm{Ac}(\tilde{a}_1(p)) \prec \mathrm{Ac}(\tilde{a}_2(p))$, then $\tilde{a}_1(p) \prec \tilde{a}_2(p)$;

iii) If $\mathrm{Ac}(\tilde{a}_1(p)) = \mathrm{Ac}(\tilde{a}_2(p))$, then $\tilde{a}_1(p) \approx \tilde{a}_2(p)$.

Definition 7. Let $S_{[0,k-1]}$ be a linguistic term set (LTS), and $\tilde{a}_1(p)$ and $\tilde{a}_2(p)$ be any two probabilistic uncertain linguistic term set fuzzy numbers (PULTSFNs), where $\#T_1=\#T_2=\#T$, $\#R_1=\#R_2=\#R$, $\#W_1=\#W_2=\#W$, and for the uncertain linguistic variable part, the function *I*(·) satisfies $I(s_\alpha) = \alpha$. Then, the Hamming distance $D_H(\tilde{a}_1(p), \tilde{a}_2(p))$ between $\tilde{a}_1(p)$ and $\tilde{a}_2(p)$ is defined as:

$$(\tilde{a}_1(p), \tilde{a}_2(p)) = \frac{1}{6}\left(\begin{array}{c}\frac{\sum_{t=1}^{\#T}\left(\begin{array}{c}\left|\left(I(s_{\mu_{1(t)}^L})\hat{p}_{1(t)}\right)^q-\left(I(s_{\mu_{2(t)}^L})\hat{p}_{2(t)}\right)^q\right|\\+\left|\left(I(s_{\mu_{1(t)}^U})\hat{p}_{1(t)}\right)^q-\left(I(s_{\mu_{2(t)}^U})\hat{p}_{2(t)}\right)^q\right|\end{array}\right)}{\#T}+\\ \frac{\sum_{r=1}^{\#R}\left(\begin{array}{c}\left|\left(I(s_{\eta_{1(r)}^L})\tilde{p}_{1(r)}\right)^q-\left(I(s_{\eta_{2(r)}^L})\tilde{p}_{2(r)}\right)^q\right|\\+\left|\left(I(s_{\eta_{1(r)}^U})\tilde{p}_{1(r)}\right)^q-\left(I(s_{\eta_{2(r)}^U})\tilde{p}_{2(r)}\right)^q\right|\end{array}\right)}{\#R}+\\ \frac{\sum_{w=1}^{\#W}\left(\begin{array}{c}\left|\left(I(s_{\nu_{1(w)}^L})\bar{p}_{1(w)}\right)^q-\left(I(s_{\nu_{2(w)}^L})\bar{p}_{2(w)}\right)^q\right|\\+\left|\left(I(s_{\nu_{1(w)}^U})\bar{p}_{1(w)}\right)^q-\left(I(s_{\nu_{2(w)}^U})\bar{p}_{2(w)}\right)^q\right|\end{array}\right)}{\#W}\end{array}\right) \tag{8}$$

3. PULTSFWA Operator

3.1 Operational Rules for the Sum of PULTSFNs

Definition 8. Let $S_{[0,k-1]}$ be a LTS, and let $\tilde{a}(p)$, $\tilde{a}_1(p)$ and $\tilde{a}_2(p)$be any three PULTSFNs, with λ>0. Then, the fundamental operational rules for the sum of these PULTSFNs are as follows:

$$(1)\ \tilde{a}_1(p)\oplus\tilde{a}_2(p) = \left(\begin{array}{c}\left\{\left[s_{k\sqrt[q]{1-\prod_{i=1}^2\left(1-\left(\mu_{i(t)}^L/k\right)^q\right)}}, s_{k\sqrt[q]{1-\prod_{i=1}^2\left(1-\left(\mu_{i(t)}^U/k\right)^q\right)}}\right]\left|\tfrac{1}{2}\textstyle\sum_{i=1}^2\hat{p}_{i(t)}\right.\right\},\\ \left\{\left[s_{k\prod_{i=1}^2\left(\eta_{i(r)}^L/k\right)}, s_{k\prod_{i=1}^2\left(\eta_{i(r)}^U/k\right)}\right]\left|\left(\textstyle\prod_{i=1}^2\tilde{p}_{i(r)}\right)^{1/2}\right.\right\},\\ \left\{\left[s_{k\prod_{i=1}^2\left(\nu_{i(w)}^L/k\right)}, s_{k\prod_{i=1}^2\left(\nu_{i(w)}^U/k\right)}\right]\left|\left(\textstyle\prod_{i=1}^2\bar{p}_{i(w)}\right)^{1/2}\right.\right\}\end{array}\right)$$

$$(2)\ \lambda(\tilde{a}(p)) = \left(\begin{array}{c}\left\{\left[s_{k\sqrt[q]{1-\left(1-\left(\mu_{(t)}^L/k\right)^q\right)^\lambda}}, s_{k\sqrt[q]{1-\left(1-\left(\mu_{(t)}^U/k\right)^q\right)^\lambda}}\right]\left|\hat{p}_{(t)}\right.\right\},\\ \left\{\left[s_{k\left(\eta_{(r)}^L/k\right)^\lambda}, s_{k\left(\eta_{(r)}^U/k\right)^\lambda}\right]\left|\tilde{p}_{(r)}\right.\right\}, \left\{\left[s_{k\left(\nu_{(w)}^L/k\right)^\lambda}, s_{k\left(\nu_{(w)}^U/k\right)^\lambda}\right]\left|\bar{p}_{(w)}\right.\right\}\end{array}\right)$$

Theorem 1. Let $\tilde{a}(p)$, $\tilde{a}_1(p)$ and $\tilde{a}_2(p)$ be any three PULTSFNs, with λ, λ_1, λ_2>0. Then,

(1) $\tilde{a}_1(p)\oplus\tilde{a}_2(p) = \tilde{a}_2(p)\oplus\tilde{a}_1(p)$;

(2) $\lambda(\tilde{a}_1(p)\oplus\tilde{a}_2(p)) = \lambda\tilde{a}_1(p)\oplus\lambda\tilde{a}_2(p)$;

(3) $\lambda_1\tilde{a}(p)\oplus\lambda_2\tilde{a}(p) = (\lambda_1+\lambda_2)\tilde{a}(p)$.

The above operational laws can be readily proven based on Definition 8.

3.2 PULTSFWA Operator

Definition 9. Let $\tilde{a}_i(p) = \langle\left\{\left[s_{\mu_{i(t)}^L}, s_{\mu_{i(t)}^U}\right]\left|\hat{p}_{i(t)}\right.\right\}, \left\{\left[s_{\eta_{i(r)}^L}, s_{\eta_{i(r)}^U}\right]\left|\tilde{p}_{i(r)}\right.\right\}, \left\{\left[s_{\nu_{i(w)}^L}, s_{\nu_{i(w)}^U}\right]\left|\bar{p}_{i(w)}\right.\right\}\rangle$(i=1,2,…,n) be a collection of PULTSFNs, and let $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ be the associated weight vector, satisfying $\omega_i > 0$ and $\sum_{i=1}^n \omega_i = 1$. Then, the probabilistic uncertain linguistic term set fuzzy weighted averaging (PULTSFWA) operator is a mapping $\Theta^n \to \Theta$ defined as:

$$PULTSFWA(\tilde{a}_1(p), \tilde{a}_2(p), \ldots, \tilde{a}_n(p)) = \oplus_{i=1}^n(\omega_i\tilde{a}_i(p)) \tag{9}$$

Theorem 2. Let $\tilde{a}_i(p)$ (*i*=1,2,…,*n*) be a collection of PULTSFNs. Then, the result obtained by aggregating via the PULTSFWA operator remains a PULTSFN, and the aggregation operator can be expressed as:

$$PULTSFWA(\tilde{a}_1(p), \tilde{a}_2(p), \ldots, \tilde{a}_n(p)) = \left(\begin{array}{c} \left\{ \left[s_{k\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(\mu_{i(t)}^{L}/k\right)^q\right)^{\omega_i}}}, s_{k\sqrt[q]{1-\prod_{i=1}^{n}\left(1-\left(\mu_{i(t)}^{U}/k\right)^q\right)^{\omega_i}}} \right] \middle| \sum_{i=1}^{n} \omega_i \, \hat{p}_{i(t)} \right\}, \\ \left\{ \left[s_{k\prod_{i=1}^{n}\left(\eta_{i(r)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{n}\left(\eta_{i(r)}^{U}/k\right)^{\omega_i}} \right] \middle| \prod_{i=1}^{n} \left(\tilde{p}_{i(r)}\right)^{\omega_i} \right\}, \\ \left\{ \left[s_{k\prod_{i=1}^{n}\left(\nu_{i(w)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{n}\left(\nu_{i(w)}^{U}/k\right)^{\omega_i}} \right] \middle| \prod_{i=1}^{n} \left(\bar{p}_{i(w)}\right)^{\omega_i} \right\} \end{array} \right) \tag{10}$$

Proof: Based on the operational rules in Definition 8, Theorem 2 can be proven using mathematical induction. For n=2, assuming $\tilde{a}_i(p)$ (i=1,2) are two PULTSFNs, then by Definition 8, we have:

$$\omega_i(\tilde{a}_i(p)) = \left(\begin{array}{c} \left\{ \left[s_{k\sqrt[q]{1-\left(1-\left(\mu_{i(t)}^{L}/k\right)^q\right)^{\omega_i}}}, s_{k\sqrt[q]{1-\left(1-\left(\mu_{i(t)}^{U}/k\right)^q\right)^{\omega_i}}} \right] \middle| \omega_i \hat{p}_{i(t)} \right\}, \\ \left\{ \left[s_{k\left(\eta_{i(r)}^{L}/k\right)^{\omega_i}}, s_{k\left(\eta_{i(r)}^{U}/k\right)^{\omega_i}} \right] \middle| \left(\tilde{p}_{i(r)}\right)^{\omega_i} \right\}, \left\{ \left[s_{k\left(\nu_{i(w)}^{L}/k\right)^{\omega_i}}, s_{k\left(\nu_{i(w)}^{U}/k\right)^{\omega_i}} \right] \middle| \left(\bar{p}_{i(w)}\right)^{\omega_i} \right\} \end{array} \right)$$

Then,

$$PULTSFWA(\tilde{a}_1(p), \tilde{a}_2(p)) = \omega_1(\tilde{a}_1(p)) \oplus \omega_2(\tilde{a}_2(p))$$

$$= \left(\begin{array}{c} \left\{ \left[s_{k\sqrt[q]{1-\left(1-\left(\mu_{1(t)}^{L}/k\right)^q\right)^{\omega_1}}}, s_{k\sqrt[q]{1-\left(1-\left(\mu_{1(t)}^{U}/k\right)^q\right)^{\omega_1}}} \right] \middle| \omega_1 \hat{p}_{1(t)} \right\}, \\ \left\{ \left[s_{k\left(\eta_{1(r)}^{L}/k\right)^{\omega_1}}, s_{k\left(\eta_{1(r)}^{U}/k\right)^{\omega_1}} \right] \middle| \left(\tilde{p}_{1(r)}\right)^{\omega_1} \right\}, \left\{ \left[s_{k\left(\nu_{1(w)}^{L}/k\right)^{\omega_1}}, s_{k\left(\nu_{1(w)}^{U}/k\right)^{\omega_1}} \right] \middle| \left(\bar{p}_{1(w)}\right)^{\omega_1} \right\} \end{array} \right)$$

$$\oplus \left(\begin{array}{c} \left\{ \left[s_{k\sqrt[q]{1-\left(1-\left(\mu_{2(t)}^{L}/k\right)^q\right)^{\omega_2}}}, s_{k\sqrt[q]{1-\left(1-\left(\mu_{2(t)}^{U}/k\right)^q\right)^{\omega_2}}} \right] \middle| \omega_2 \hat{p}_{2(t)} \right\}, \\ \left\{ \left[s_{k\left(\eta_{2(r)}^{L}/k\right)^{\omega_2}}, s_{k\left(\eta_{2(r)}^{U}/k\right)^{\omega_2}} \right] \middle| \left(\tilde{p}_{2(r)}\right)^{\omega_2} \right\}, \left\{ \left[s_{k\left(\nu_{2(w)}^{L}/k\right)^{\omega_2}}, s_{k\left(\nu_{2(w)}^{U}/k\right)^{\omega_2}} \right] \middle| \left(\bar{p}_{2(w)}\right)^{\omega_2} \right\} \end{array} \right)$$

$$= \left(\begin{array}{c} \left\{ \left[s_{k\sqrt[q]{1-\prod_{i=1}^{2}\left(1-\left(\mu_{i(t)}^{L}/k\right)^q\right)^{\omega_i}}}, s_{k\sqrt[q]{1-\prod_{i=1}^{2}\left(1-\left(\mu_{i(t)}^{U}/k\right)^q\right)^{\omega_i}}} \right] \middle| \sum_{i=1}^{2} \omega_i \, \hat{p}_{i(t)} \right\}, \\ \left\{ \left[s_{k\prod_{i=1}^{2}\left(\eta_{i(r)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{2}\left(\eta_{i(r)}^{U}/k\right)^{\omega_i}} \right] \middle| \prod_{i=1}^{2} \left(\tilde{p}_{i(r)}\right)^{\omega_i} \right\}, \\ \left\{ \left[s_{k\prod_{i=1}^{2}\left(\nu_{i(w)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{2}\left(\nu_{i(w)}^{U}/k\right)^{\omega_i}} \right] \middle| \prod_{i=1}^{2} \left(\bar{p}_{i(w)}\right)^{\omega_i} \right\} \end{array} \right)$$

Therefore, the result is also valid for n=2.

Assuming that the result holds for n =θ, then Equation (11) holds, i.e.,

$$PULTSFWA(\tilde{a}_1(p),\tilde{a}_2(p),\dots,\tilde{a}_\theta(p)) = \left(\begin{array}{c}\left\{\left[s_{k\sqrt[q]{1-\prod_{i=1}^{\theta}\left(1-\left(\mu_{i(t)}^{L}/k\right)^{q}\right)^{\omega_i}}}, s_{k\sqrt[q]{1-\prod_{i=1}^{\theta}\left(1-\left(\mu_{i(t)}^{U}/k\right)^{q}\right)^{\omega_i}}}\right]\middle|\sum_{i=1}^{\theta}\omega_i\hat{p}_{i(t)}\right\},\\ \left\{\left[s_{k\prod_{i=1}^{\theta}\left(\eta_{i(r)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{\theta}\left(\eta_{i(r)}^{U}/k\right)^{\omega_i}}\right]\middle|\prod_{i=1}^{\theta}\left(\tilde{p}_{i(r)}\right)^{\omega_i}\right\},\\ \left\{\left[s_{k\prod_{i=1}^{\theta}\left(\nu_{i(w)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{\theta}\left(\nu_{i(w)}^{U}/k\right)^{\omega_i}}\right]\middle|\prod_{i=1}^{\theta}\left(\bar{p}_{i(w)}\right)^{\omega_i}\right\}\end{array}\right)$$

Then, for $n=\theta+1$, according to the operational rules in Definition 8, we obtain:

$$PULTSFWA(\tilde{a}_1(p),\tilde{a}_2(p),\dots,\tilde{a}_\theta(p),\tilde{a}_{\theta+1}(p))$$

$$= \left(\begin{array}{c}\left\{\left[s_{k\sqrt[q]{1-\prod_{i=1}^{\theta}\left(1-\left(\mu_{i(t)}^{L}/k\right)^{q}\right)^{\omega_i}}}, s_{k\sqrt[q]{1-\prod_{i=1}^{\theta}\left(1-\left(\mu_{i(t)}^{U}/k\right)^{q}\right)^{\omega_i}}}\right]\middle|\sum_{i=1}^{\theta}\omega_i\hat{p}_{i(t)}\right\},\\ \left\{\left[s_{k\prod_{i=1}^{\theta}\left(\eta_{i(r)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{\theta}\left(\eta_{i(r)}^{U}/k\right)^{\omega_i}}\right]\middle|\prod_{i=1}^{\theta}\left(\tilde{p}_{i(r)}\right)^{\omega_i}\right\},\\ \left\{\left[s_{k\prod_{i=1}^{\theta}\left(\nu_{i(w)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{\theta}\left(\nu_{i(w)}^{U}/k\right)^{\omega_i}}\right]\middle|\prod_{i=1}^{\theta}\left(\bar{p}_{i(w)}\right)^{\omega_i}\right\}\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\left\{\left[s_{k\sqrt[q]{1-\left(1-\left(\mu_{\theta+1(t)}^{L}/k\right)^{q}\right)^{\omega_{\theta+1}}}}, s_{k\sqrt[q]{1-\left(1-\left(\mu_{\theta+1(t)}^{U}/k\right)^{q}\right)^{\omega_{\theta+1}}}}\right]\middle|\omega_{\theta+1}\hat{p}_{\theta+1(t)}\right\},\\ \left\{\left[s_{k\left(\eta_{\theta+1(r)}^{L}/k\right)^{\omega_{\theta+1}}}, s_{k\left(\eta_{\theta+1(r)}^{U}/k\right)^{\omega_{\theta+1}}}\right]\middle|\left(\tilde{p}_{\theta+1(r)}\right)^{\omega_{\theta+1}}\right\},\\ \left\{\left[s_{k\left(\nu_{\theta+1(w)}^{L}/k\right)^{\omega_{\theta+1}}}, s_{k\left(\nu_{\theta+1(w)}^{U}/k\right)^{\omega_{\theta+1}}}\right]\middle|\left(\bar{p}_{\theta+1(w)}\right)^{\omega_{\theta+1}}\right\}\end{array}\right)$$

$$= \left(\begin{array}{c}\left\{\left[s_{k\sqrt[q]{1-\prod_{i=1}^{\theta+1}\left(1-\left(\mu_{i(t)}^{L}/k\right)^{q}\right)^{\omega_i}}}, s_{k\sqrt[q]{1-\prod_{i=1}^{\theta+1}\left(1-\left(\mu_{i(t)}^{U}/k\right)^{q}\right)^{\omega_i}}}\right]\middle|\sum_{i=1}^{\theta+1}\omega_i\hat{p}_{i(t)}\right\},\\ \left\{\left[s_{k\prod_{i=1}^{\theta+1}\left(\eta_{i(r)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{\theta+1}\left(\eta_{i(r)}^{U}/k\right)^{\omega_i}}\right]\middle|\prod_{i=1}^{\theta+1}\left(\tilde{p}_{i(r)}\right)^{\omega_i}\right\},\\ \left\{\left[s_{k\prod_{i=1}^{\theta+1}\left(\nu_{i(w)}^{L}/k\right)^{\omega_i}}, s_{k\prod_{i=1}^{\theta+1}\left(\nu_{i(w)}^{U}/k\right)^{\omega_i}}\right]\middle|\prod_{i=1}^{\theta+1}\left(\bar{p}_{i(w)}\right)^{\omega_i}\right\}\end{array}\right)$$

Obviously, this equation is also valid for $n=\theta+1$. Therefore, Equation (11) holds for all positive integers n.

Below are the fundamental properties of the PULTSFWA operator:

Theorem 3. (Monotonicity) Let $\tilde{a}_i(p) = \langle\left\{\left[s_{\mu_{i(t)}^{L}}, s_{\mu_{i(t)}^{U}}\right]|\hat{p}_{i(t)}\right\},\left\{\left[s_{\eta_{i(r)}^{L}}, s_{\eta_{i(r)}^{U}}\right]|\tilde{p}_{i(r)}\right\},\left\{\left[s_{\nu_{i(w)}^{L}}, s_{\nu_{i(w)}^{U}}\right]|\bar{p}_{i(w)}\right\}\rangle$ and $\tilde{a}_i'(p) = \langle\left\{\left[s_{\mu_{i(t)}^{\prime L}}, s_{\mu_{i(t)}^{\prime U}}\right]|\widehat{p'}_{i(t)}\right\},\left\{\left[s_{\eta_{i(r)}^{\prime L}}, s_{\eta_{i(r)}^{\prime U}}\right]|\widetilde{p'}_{i(r)}\right\},\left\{\left[s_{\nu_{i(w)}^{\prime L}}, s_{\nu_{i(w)}^{\prime U}}\right]|\overline{p'}_{i(w)}\right\}\rangle$ (i=1,2,…,n) be two collections of PULTSFNs. If for all i, the following conditions hold: $\mu_{i(t)}^{U}\hat{p}_{i(t)} \le \mu_{i(t)}^{\prime U}\widehat{p'}_{i(t)}$, $\eta_{i(r)}^{U}\tilde{p}_{i(r)} \ge \eta_{i(r)}^{\prime U}\widetilde{p'}_{i(r)}$ and $\nu_{i(w)}^{U}\bar{p}_{i(w)} \ge \nu_{i(w)}^{\prime U}\overline{p'}_{i(w)}$, then

$$PULTSFWA(\tilde{a}_1(p),\tilde{a}_2(p),\dots,\tilde{a}_n(p)) \le PULTSFWA(\tilde{a}_1'(p),\tilde{a}_2'(p),\dots,\tilde{a}_n'(p)) \tag{11}$$

Theorem 4. (Idempotency) If $\tilde{a}_i(p) = \langle\left\{\left[s_{\mu_{i(t)}^{L}}, s_{\mu_{i(t)}^{U}}\right]|\hat{p}_{i(t)}\right\},\left\{\left[s_{\eta_{i(r)}^{L}}, s_{\eta_{i(r)}^{U}}\right]|\tilde{p}_{i(r)}\right\},\left\{\left[s_{\nu_{i(w)}^{L}}, s_{\nu_{i(w)}^{U}}\right]|\bar{p}_{i(w)}\right\}\rangle$ (i=1,2,…,n) are all equal, i.e., $\tilde{a}_i(p) = \tilde{a}(p)$ for all i, then

$$PULTSFWA(\tilde{a}_1(p),\tilde{a}_2(p),\dots,\tilde{a}_n(p)) = \tilde{a}(p) \tag{12}$$

Theorem 5. (Boundedness) Let $\tilde{a}_i(p)$ (i=1,2,…,n) be a collection of PULTSFNs. Then,

$$\tilde{d}^{-} \leq PULTSFWA(\tilde{a}_1(p), \tilde{a}_2(p), \ldots, \tilde{a}_n(p)) \leq \tilde{d}^{+} \tag{13}$$

where $\tilde{d}^{+}$ is the PULTSFN corresponding to $\max_i Sc(\tilde{a}_i(p))$, and $\tilde{d}^{-}$ is the PULTSFN corresponding to $\min_i Sc(\tilde{a}_i(p))$.

4. Decision-Making Model Based on PULTSF ARAS

In a typical MCDM problem, decision-makers evaluate a finite set of alternatives: $\mathcal{H} = \{h_i | i = 1,2,\ldots,m\}$, with respect to a corresponding set of attributes: $\aleph = \{\beth_j | j = 1,2,\ldots,n\}$. Each attribute is assigned a relative importance represented by a weight vector: $\boldsymbol{\varpi} = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$, where $\varpi_j \geq 0$ and $\sum_{j=1}^{n} \varpi_j = 1$. Experts provide evaluation information for each alternative under each attribute using PULTSFNs. These evaluations are organized into a PULTSF decision matrix $\widetilde{\mathcal{D}} = [\tilde{d}_{ij}(p)]_{m\times n}$, where $\tilde{d}_{ij}(p) = \langle\{[s_{\mu^L_{ij(t)}}, s_{\mu^U_{ij(t)}}] | \hat{p}_{ij(t)}\}, \{[s_{\eta^L_{ij(r)}}, s_{\eta^U_{ij(r)}}] | \tilde{p}_{ij(r)}\}, \{[s_{\nu^L_{ij(w)}}, s_{\nu^U_{ij(w)}}] | \bar{p}_{ij(w)}\}\rangle$ (*i*=1, 2,...,*m*; *j*=1, 2,...,*n*; *t*=1, 2,...,#*T*; *r*=1, 2, ...,#*R*; *w*=1, 2, ...,#*W*).

Step 1. Based on the initial PULTSF decision matrix $\widetilde{\mathcal{D}}$, utilize Definition 5 to normalize the PULTSFNs. Subsequently, apply Equation (14) to transform the attribute values of cost-type and benefit-type attributes, thereby achieving the standardization of attribute evaluation information within the PULTSF context. Consequently, the standardized PULTSF decision matrix $\tilde{\mathcal{R}} = [\tilde{r}_{ij}(p)]_{m\times n}$ is obtained.

$$\tilde{r}_{ij}(p) = \begin{cases} \tilde{d}_{ij}(p) = \left(\left\{\begin{bmatrix} s_{\mu^L_{ij(t)}}, \\ s_{\mu^U_{ij(t)}} \end{bmatrix} \middle| \hat{p}_{ij(t)}\right\}, \left\{\begin{bmatrix} s_{\eta^L_{ij(r)}}, \\ s_{\eta^U_{ij(r)}} \end{bmatrix} \middle| \tilde{p}_{ij(r)}\right\}, \left\{\begin{bmatrix} s_{\nu^L_{ij(w)}}, \\ s_{\nu^U_{ij(w)}} \end{bmatrix} \middle| \bar{p}_{ij(w)}\right\}\right), j \in J_1 \\ (\tilde{d}_{ij}(p))^c = \left(\left\{\begin{bmatrix} s_{\nu^L_{ij(w)}}, \\ s_{\nu^U_{ij(w)}} \end{bmatrix} \middle| \bar{p}_{ij(w)}\right\}, \left\{\begin{bmatrix} s_{\eta^L_{ij(r)}}, \\ s_{\eta^U_{ij(r)}} \end{bmatrix} \middle| \tilde{p}_{ij(r)}\right\}, \left\{\begin{bmatrix} s_{\mu^L_{ij(t)}}, \\ s_{\mu^U_{ij(t)}} \end{bmatrix} \middle| \hat{p}_{ij(t)}\right\}\right), j \in J_2 \end{cases} \tag{14}$$

where J_1 and J_2 represent benefit-type and cost-type attributes, respectively.

Step 2. Calculate the attribute weights.

Step 2.1 Calculate the distance between PULTSFNs $\tilde{r}_{kj}(p)$ and $\tilde{r}_{lj}(p)$ under attribute $\beth_j$ based on the proposed PULTSF Hamming distance $D_H(\tilde{r}_{ij}(p), \tilde{r}_{lj}(p))$(*l*<*i*, *i*, *l*=1, 2,..., *m*).

Step 2.2 Calculate the average distance C_j among the PULTSFNs $\tilde{r}_{kj}(p)$ and $\tilde{r}_{lj}(p)$ under attribute $\beth_j$ using the following formula:

$$C_j = \frac{2}{m(m-1)} \sum_{i=1}^{m} \sum_{l=1}^{m} D_H(\tilde{r}_{ij}(p), \tilde{r}_{lj}(p)) \tag{15}$$

Step 2.3 Calculate the comprehensive weight w_j^c of the attribute as shown in Equation (16), where w_j^s represents the subjective weight value assigned by experts to attribute $\beth_j$.

$$w_j^c = \frac{w_j^s \times C_j}{\sum_{j=1}^{n} w_j^s} \tag{16}$$

Step 2.4 Calculate the normalized weight w_j of the attribute, satisfying $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

$$w_j = \frac{w_j^c}{\sum_{j=1}^{n} w_j^c} \tag{17}$$

Step 3. Determine the PULTSF positive ideal solution (PIS) from the standardized PULTSF decision matrix $\tilde{\mathcal{R}} = [\tilde{r}_{ij}(p)]_{m\times n}$, denoted as $\tilde{r}_{+} = \{\tilde{r}_1^{+}(p), \tilde{r}_2^{+}(p), \cdots, \tilde{r}_j^{+}(p), \cdots, \tilde{r}_n^{+}(p)\}$, where $\tilde{r}_j^{+}(p) = \tilde{r}_{ij}(p)$ and $i = F_{max}\left(Sc(\tilde{r}_{ij}(p))\right)$ for each i. Here, $F_{max}(\cdot)$ is a function that returns the value of i that maximizes $Sc(\tilde{r}_{ij}(p))$.

Step 4. Aggregate the PULTSFNs under each attribute for the *i*-th alternative using the PULTSFWA operator to obtain the comprehensive value $\tilde{Q}_i(p)$ of that alternative. Similarly, aggregate the PULTSFNs under each attribute of the PIS using the PULTSFWA operator to obtain the comprehensive value $\tilde{Q}^{+}(p)$ of the PIS.

Step 5. Calculate the degree of proximity between each alternative and the PIS using Equation (18).

$$\delta(h_i) = D_H\left(\tilde{Q}_i(p), \tilde{Q}^{+}(p)\right) \tag{18}$$

Step 6. Rank the alternatives in ascending order according to the $\delta(h_i)$ values, and determine the most desirable alternative. A smaller $\delta(h_i)$ value indicates a closer proximity to the PIS, signifying that the alternative h_i is better.

4. Numerical Example

A company, a well-known manufacturer of new energy vehicles and related components, is currently advancing a company-wide green transformation. As part of this transition, the company is reassessing its green suppliers to support its operations. The company conducted a comprehensive screening process, which included qualification reviews, corporate credit assessments, and other related evaluations. Through this screening, four green suppliers $\mathcal{H} = \{h_1, h_2, h_3, h_4\}$ were identified for the expert evaluation and selection stage.

The task of the expert panel is to select the most suitable green supplier by evaluating the corporate products and leveraging their own knowledge and experience. During the decision-making process, the expert panel customized four key evaluation criteria: ב$_1$: Quality; ב$_2$: Cost; ב$_3$: Service Level; ב$_4$: Production Capacity. Among these, ב$_2$ is a cost-type criterion, while the others are benefit-type criteria. Based on the consensus reached through expert discussions, the subjective weight vector for these criteria is given as w^s=(0.4, 0.3, 0.1, 0.2)^T. The company's core objective is to prioritize the four candidate green suppliers and select the best one with the help of the expert panel's professional knowledge.

To accurately capture the information in the evaluation of the four criteria, the linguistic term set $S_{[0,6]} = \{s_0, s_1, \cdots, s_6\}$ ={*Very Low, Low, Slightly Low, General, Slightly High, High, Very High*} is utilized. The expert panel is authorized to employ probabilistic uncertain linguistic term set fuzzy numbers (PULTSFNs) to describe the evaluation information. The evaluation data provided by the expert panel are presented in Table 1.

Table 1
Evaluation information provided by experts

Options	ב$_1$	ב$_2$	ב$_3$	ב$_4$
h_1	$\langle\{[s_2,s_3]\vert 0.5,[s_3,s_4]\vert 0.5\}$, $\{[s_1,s_2]\vert 0.3,[s_2,s_3]\vert 0.7\}$, $\{[s_2,s_3]\vert 0.5,[s_3,s_4]\vert 0.5\}\rangle$	$\langle\{s_3\vert 0.3,[s_3,s_4]\vert 0.7\}$, $\{[s_1,s_2]\vert 0.6,[s_2,s_3]\vert 0.4\}$, $\{[s_1,s_2]\vert 0.2,[s_2,s_3]\vert 0.8\}\rangle$	$\langle\{[s_4,s_5]\vert 0.7,[s_5,s_6]\vert 0.3\}$ $\{[s_2,s_3]\vert 0.5,[s_3,s_4]\vert 0.5\}$, $\{[s_1,s_2]\vert 0.6,[s_2,s_3]\vert 0.4\}\rangle$	$\langle\{[s_3,s_4]\vert 0.4,[s_4,s_5]\vert 0.6\}$ $\{[s_0,s_1]\vert 0.7,[s_1,s_2]\vert 0.3\}$, $\{[s_2,s_3]\vert 0.6,[s_3,s_4]\vert 0.4\}\rangle$
h_2	$\langle\{[s_3,s_4]\vert 1\}$, $\{[s_0,s_1]\vert 0.2,[s_1,s_2]\vert 0.8\}$, $\{[s_3,s_4]\vert 0.3,[s_4,s_5]\vert 0.7\}\rangle$	$\langle\{[s_3,s_4]\vert 0.2,[s_4,s_5]\vert 0.8\}$ $\{[s_2,s_3]\vert 0.6,[s_3,s_4]\vert 0.4\}$, $\{[s_2,s_3]\vert 0.3,[s_3,s_4]\vert 0.7\}\rangle$	$\langle\{s_3\vert 0.7,[s_3,s_4]\vert 0.3\}$, $\{[s_1,s_2]\vert 0.6,[s_2,s_3]\vert 0.4\}$, $\{[s_2,s_3]\vert 0.5,[s_3,s_4]\vert 0.5\}\rangle$	$\langle\{[s_1,s_2]\vert 0.3,[s_2,s_3]\vert 0.7\}$ $\{[s_1,s_2]\vert 0.5,[s_2,s_3]\vert 0.5\}$, $\{[s_3,s_4]\vert 0.8,[s_4,s_5]\vert 0.2\}\rangle$
h_3	$\langle\{[s_3,s_4]\vert 0.4,[s_4,s_5]\vert 0.6\}$, $\{[s_1,s_2]\vert 0.5,[s_2,s_3]\vert 0.5\}$, $\{[s_1,s_2]\vert 0.7,[s_2,s_3]\vert 0.3\}\rangle$	$\langle\{[s_3,s_4]\vert 0.7,[s_4,s_5]\vert 0.3\}$ $\{[s_0,s_1]\vert 0.8,[s_1,s_2]\vert 0.2\}$, $\{[s_1,s_2]\vert 0.6,[s_2,s_3]\vert 0.4\}\rangle$	$\langle\{[s_1,s_2]\vert 0.3,[s_2,s_3]\vert 0.7\}$, $\{[s_1,s_2]\vert 0.8,[s_2,s_3]\vert 0.2\}$, $\{[s_3,s_4]\vert 0.4,[s_4,s_5]\vert 0.6\}\rangle$	$\langle\{[s_2,s_3]\vert 0.6,[s_3,s_4]\vert 0.4\}$, $\{[s_0,s_1]\vert 0.7,[s_1,s_2]\vert 0.3\}$, $\{[s_2,s_3]\vert 0.4,[s_3,s_4]\vert 0.6\}\rangle$
h_4	$\langle\{[s_2,s_3]\vert 0.6,[s_3,s_4]\vert 0.4\}$, $\{[s_1,s_2]\vert 0.6,[s_2,s_3]\vert 0.4\}$, $\{[s_3,s_4]\vert 0.5,s_4\vert 0.5\}\rangle$	$\langle\{[s_0,s_1]\vert 0.4,[s_1,s_2]\vert 0.6\}$ $\{[s_1,s_2]\vert 0.6,[s_2,s_3]\vert 0.4\}$, $\{[s_3,s_4]\vert 0.5,[s_4,s_5]\vert 0.5\}\rangle$	$\langle\{s_4\vert 0.4,[s_4,s_5]\vert 0.6\}$, $\{[s_1,s_2]\vert 0.7,[s_2,s_3]\vert 0.3\}$, $\{[s_2,s_3]\vert 0.5,[s_3,s_4]\vert 0.5\}\rangle$	$\langle\{[s_2,s_3]\vert 0.3,[s_3,s_4]\vert 0.7\}$ $\{[s_1,s_2]\vert 0.4,[s_2,s_3]\vert 0.6\}$, $\{[s_1,s_2]\vert 0.8,[s_2,s_3]\vert 0.2\}\rangle$

The specific steps are as follows:

Step 1: Utilizing Definition 6 and Formula (14), the standardized PULTSF decision matrix $\tilde{\mathcal{R}}$ is obtained, as shown in Table 2.

Step 2: Using Formulas (15) to (17), the comprehensive weight values for each criterion are calculated as follows: $w_1 = 0.345$, $w_2 = 0.378$, $w_3 = 0.110$, $w_4 = 0.167$.

Step 3: Based on the comparison of score function values, the PIS is determined as follows:

$$PIS = \begin{pmatrix} \beth_1: & \langle\{[s_3,s_4]|0.4,[s_4,s_5]|0.6\},\{[s_1,s_2]|0.5,[s_2,s_3]|0.5\},\{[s_1,s_2]|0.7,[s_2,s_3]|0.3\}\rangle \\ \beth_2: & \langle\{[s_3,s_4]|0.5,[s_4,s_5]|0.5\},\{[s_1,s_2]|0.6,[s_2,s_3]|0.4\},\{[s_0,s_1]|0.4,[s_1,s_2]|0.6\}\rangle \\ \beth_3: & \langle\{[s_4,s_5]|0.7,[s_5,s_6]|0.3\},\{[s_2,s_3]|0.5,[s_3,s_4]|0.5\},\{[s_1,s_2]|0.6,[s_2,s_3]|0.4\}\rangle \\ \beth_4: & \langle\{[s_3,s_4]|0.4,[s_4,s_5]|0.6\},\{[s_0,s_1]|0.7,[s_1,s_2]|0.3\},\{[s_2,s_3]|0.6,[s_3,s_4]|0.4\}\rangle \end{pmatrix}$$

Step 4: Using the PULTSFWA operator to aggregate the PULTSFNs under each attribute for the *i*-th alternative, the comprehensive value $\tilde{Q}_i(p)$ for each alternative is obtained, as shown in Table 3.

Table 2
Standardized PULTSF decision matrix

Options	$\beth_1$	$\beth_2$	$\beth_3$	$\beth_4$
h_1	$<\{[s_2,s_3]\vert\vert 0.5,[s_3,s_4]\vert\vert 0.5\},$ $\{[s_1,s_2]\vert\vert 0.3,[s_2,s_3]\vert\vert 0.7\},$ $\{[s2,s3]\vert\vert 0.5,[s3,s4]\vert\vert 0.5\}>$	$<\{[s_1,s_2]\vert\vert 0.2,[s_2,s_3]\vert\vert 0.8\},$ $\{[s_1,s_2]\vert\vert 0.6,[s_2,s_3]\vert\vert 0.4\},$ $\{[s_3,s_3]\vert\vert 0.3,[s_3,s_4]\vert\vert 0.7\}>$	$<\{[s_4,s_5]\vert\vert 0.7,[s_5,s_6]\vert\vert 0.3\},$ $\{[s_2,s_3]\vert\vert 0.5,[s_3,s_4]\vert\vert 0.5\},$ $\{[s_1,s_2]\vert\vert 0.6,[s_2,s_3]\vert\vert 0.4\}>$	$<\{[s_3,s_4]\vert\vert 0.4,[s_4,s_5]\vert\vert 0.6\},$ $\{[s_0,s_1]\vert\vert 0.7,[s_1,s_2]\vert\vert 0.3\},$ $\{[s_2,s_3]\vert\vert 0.6,[s_3,s_4]\vert\vert 0.4\}>$
h_2	$<\{[s_3,s_4]\vert\vert 0.5,[s_3,s_4]\vert\vert 0.5\},$ $\{[s_0,s_1]\vert\vert 0.2,[s_1,s_2]\vert\vert 0.8\},$ $\{[s_3,s_4]\vert\vert 0.3,[s_4,s_5]\vert\vert 0.7\}>$	$<\{[s_2,s_3]\vert\vert 0.3,[s_3,s_4]\vert\vert 0.7\},$ $\{[s_2,s_3]\vert\vert 0.6,[s_3,s_4]\vert\vert 0.4\},$ $\{[s_3,s_4]\vert\vert 0.2,[s_4,s_5]\vert\vert 0.8\}>$	$<\{[s_3,s_3]\vert\vert 0.7,[s_3,s_4]\vert\vert 0.3\},$ $\{[s_1,s_2]\vert\vert 0.6,[s_2,s_3]\vert\vert 0.4\},$ $\{[s_2,s_3]\vert\vert 0.5,[s_3,s_4]\vert\vert 0.5\}>$	$<\{[s_1,s_2]\vert\vert 0.3,[s_2,s_3]\vert\vert 0.7\},$ $\{[s_1,s_2]\vert\vert 0.5,[s_2,s_3]\vert\vert 0.5\},$ $\{[s_3,s_4]\vert\vert 0.8,[s_4,s_5]\vert\vert 0.2\}>$
h_3	$<\{[s_3,s_4]\vert\vert 0.4,[s_4,s_5]\vert\vert 0.6\},$ $\{[s_1,s_2]\vert\vert 0.5,[s_2,s_3]\vert\vert 0.5\},$ $\{[s_1,s_2]\vert\vert 0.7,[s_2,s_3]\vert\vert 0.3\}>$	$<\{[s_1,s_2]\vert\vert 0.6,[s_2,s_3]\vert\vert 0.4\},$ $\{[s_0,s_1]\vert\vert 0.8,[s_1,s_2]\vert\vert 0.2\},$ $\{[s_3,s_4]\vert\vert 0.7,[s_4,s_5]\vert\vert 0.3\}>$	$<\{[s_1,s_2]\vert\vert 0.3,[s_2,s_3]\vert\vert 0.7\},$ $\{[s_1,s_2]\vert\vert 0.8,[s_2,s_3]\vert\vert 0.2\},$ $\{[s_3,s_4]\vert\vert 0.4,[s_4,s_5]\vert\vert 0.6\}>$	$<\{[s_2,s_3]\vert\vert 0.6,[s_3,s_4]\vert\vert 0.4\},$ $\{[s_0,s_1]\vert\vert 0.7,[s_1,s_2]\vert\vert 0.3\},$ $\{[s_2,s_3]\vert\vert 0.4,[s_3,s_4]\vert\vert 0.6\}>$
h_4	$<\{[s_2,s_3]\vert\vert 0.6,[s_3,s_4]\vert\vert 0.4\},$ $\{[s_1,s_2]\vert\vert 0.6,[s_2,s_3]\vert\vert 0.4\},$ $\{[s_3,s_4]\vert\vert 0.5,[s_4,s_4]\vert\vert 0.5\}>$	$<\{[s_3,s_4]\vert\vert 0.5,[s_4,s_5]\vert\vert 0.5\},$ $\{[s_1,s_2]\vert\vert 0.6,[s_2,s_3]\vert\vert 0.4\},$ $\{[s_0,s_1]\vert\vert 0.4,[s_1,s_2]\vert\vert 0.6\}>$	$<\{[s_4,s_4]\vert\vert 0.4,[s_4,s_5]\vert\vert 0.6\},$ $\{[s_1,s_2]\vert\vert 0.7,[s_2,s_3]\vert\vert 0.3\},$ $\{[s_2,s_3]\vert\vert 0.5,[s_3,s_4]\vert\vert 0.5\}>$	$<\{[s_2,s_3]\vert\vert 0.3,[s_3,s_4]\vert\vert 0.7\},$ $\{[s_1,s_2]\vert\vert 0.4,[s_2,s_3]\vert\vert 0.6\},$ $\{[s_1,s_2]\vert\vert 0.8,[s_2,s_3]\vert\vert 0.2\}>$

Table 3
Calculation results

Options	$\tilde{Q}_i(p)$	$\delta(h_i)$
h_1	$<\{[s_{2.483},s_{3.399}]\vert\vert 0.392,[s_{3.399},s_{4.408}]\vert\vert 0.608\},\{[s_{0.000},s_{1.863}]\vert\vert 0.475[s_{1.863},s_{2.893}]\vert\vert 0.474\},$ $\{[s_{2.160},s_{2.869}]\vert\vert 0.434,[s_{2.869},s_{3.875}]\vert\vert 0.534\}>$	2.037
h_2	$<\{[s_{2.502},s_{3.342}]\vert\vert 0.413,[s_{2.995},s_{3.945}]\vert\vert 0.587\},\{[s_{0.000},s_{1.835}]\vert\vert 0.398,[s_{1.835},s_{2.908}]\vert\vert 0.527\},$ $\{[s_{2.869},s_{3.875}]\vert\vert 0.321,[s_{3.875},s_{4.878}]\vert\vert 0.576\}>$	4.815
h_3	$<\{[s_{2.247},s_{3.159}]\vert\vert 0.498,[s_{3.159},s_{4.135}]\vert\vert 0.502\},\{[s_{0.000},s_{1.317}]\vert\vert 0.665,[s_{1.317},s_{2.405}]\vert\vert 0.294\},$ $\{[s_{1.919},s_{3.002}]\vert\vert 0.599,[s_{3.002},s_{4.039}]\vert\vert 0.364\}>$	2.922
h_4	$<\{[s_{2.795},s_{3.573}]\vert\vert 0.490,[s_{3.573},s_{4.573}]\vert\vert 0.510\},\{[s_{1.000},s_{2.000}]\vert\vert 0.570,[s_{2.000},s_{3.000}]\vert\vert 0.415\},$ $\{[s_{0.000},s_{2.044}]\vert\vert 0.497,[s_{2.044},s_{2.934}]\vert\vert 0.460\}>$	1.460
PIS	$<\{[s_{3.151},s_{4.150}]\vert\vert 0.471,[s_{4.150},s_{5.162}]\vert\vert 0.529\},\{[s_{0.000},s_{1.865}]\vert\vert 0.567,[s_{1.865},s_{2.894}]\vert\vert 0.422\},$ $\{[s_{0.000},s_{1.647}]\vert\vert 0.543,[s_{1.647},s_{2.700}]\vert\vert 0.422\}>$	-

Step 5: Using Formula (18), the proximity degree $\delta(h_i)$ between each alternative and the PIS is calculated, as shown in Table 3.

Step 6: From the results in Table 3, the ranking of the alternatives is: $h_4 \succ h_1 \succ h_3 \succ h_2$. Therefore, alternative h_4 is the optimal choice.

To test the influence of parameter q variation on the ranking results of alternatives, the parameter q is assigned values within the range [3, 7]. The utility values of each alternative and their corresponding rankings are presented in Table 4 and Figure 1.

Table 4
Utility values of alternatives with respect to *q* variation

q	3	3.5	4	4.5	5	5.5	6	6.5	7
h_1	2.037	2.795	3.865	5.844	9.863	17.958	34.075	66.600	132.771
h_2	4815	7.144	13.119	21.980	37.284	64.171	112.111	199.074	359.552
h_3	2.922	4.473	6.910	10.773	16.930	26.787	42.605	68.031	108.937
h_4	1.460	2.377	3.871	6.301	10.256	16.664	27.025	43.748	70.836

Figure 1 illustrates that as the value of q increases, the utility values of all alternatives also increase, while the ranking of the alternatives changes accordingly. For example, when q=3, the ranking of alternatives is $h_4 \succ h_1 \succ h_3 \succ h_2$; when q=4, the ranking becomes $h_1 \succ h_4 \succ h_3 \succ h_2$; and when q=7, the ranking shifts to $h_4 \succ h_3 \succ h_1 \succ h_2$. The reason for this lies in the fact that variations in the parameter q not only affect the expression of the commitment degree and hesitation degree of PULTSFNs but also influence the weights of the criteria.

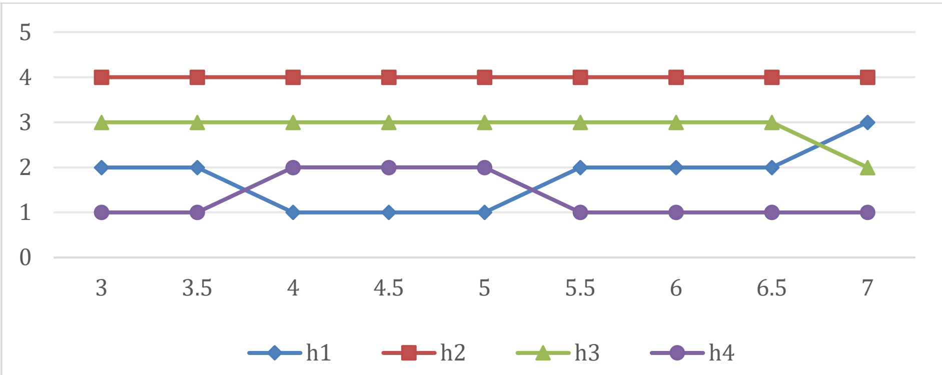

Fig. 1 Variation of alternative rankings with respect to *q*

As shown in Figure 2, with an increase in *q*, the weight values of criteria $\beth_1$, $\beth_3$, and $\beth_4$ decrease, while the weight value of criterion $\beth_2$ increases. Consequently, changes in the parameter *q* lead to variations in the ranking of alternatives. This demonstrates that the proposed method can flexibly select an appropriate q value based on different decision-making scenarios to obtain the optimal alternative.

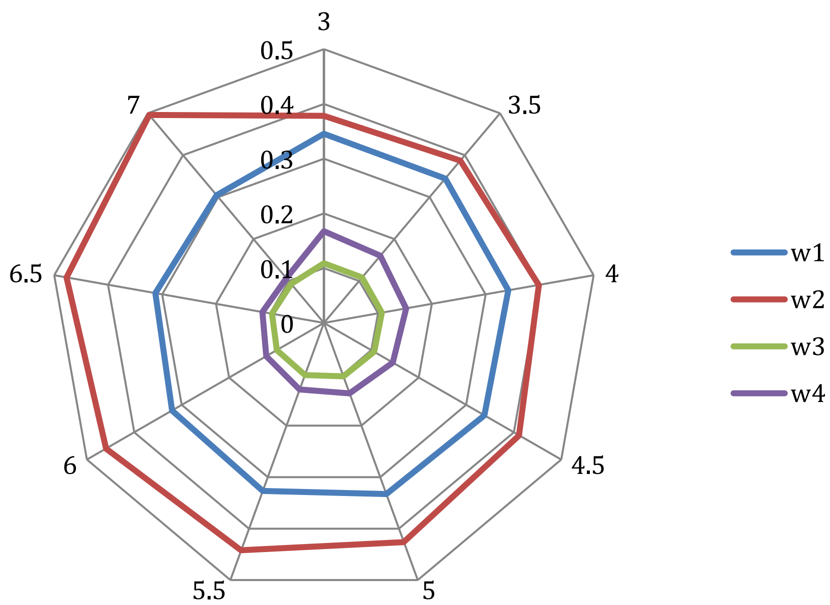

Fig. 2 Variation of criterion weights with respect to *q*

PULTSFS possesses a generalized nature and can, under certain scenarios, degenerate into specialized decision-making environments, such as PULqROFS [16], PULIFS [15], PLTSFS [26,28], PLq-ROFS [29], PULTS [12], etc. However, the methods applicable in these decision-making environments cannot be directly applied to evaluation data within the PULTSFS framework. Furthermore, some existing aggregation operators lack closure compared to the operator proposed in this paper. For example, the PULTS operational rules in [12] convert PULVs into uncertain linguistic numbers, a process that may result in the loss of partial information during aggregation. In the normalization process of some existing aggregation operators [26], the probability distribution is fixed such that probability values do not participate in the aggregation process. This approach becomes more operationally complex and less practical when addressing real-world problems. Additionally, in [26], the aggregation process adopts a multiplicative approach for probability values, rendering the final probability distribution unrealistic. Therefore, in comparison, the method proposed in this paper demonstrates stronger advantages.

5. Conclusions

Building upon PULTS and TSFS, this paper proposes a novel concept of PULTSFS, introduces relevant fundamental definitions, including score functions, accuracy functions, comparison rules, and the PULTSF Hamming distance. Furthermore, based on the basic operational rules of PULTSFNs, the PULTSFWA operator is developed, and its related properties are discussed. Within this decision-making environment, the ARAS method is extended, and a decision-making model is constructed. The effectiveness and feasibility of the proposed method are validated through a numerical example analysis. However, the proposed method also has certain limitations, such as neglecting the interrelationships among criteria and presupposing subjective weights for the criteria. In future research, efforts will focus on addressing these shortcomings. Additionally, consensus issues in group decision-making problems will also be a key area of study in the theory and methodology of PULTSFS decision-making.

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Conflicts of Interest

The authors declare no conflicts of interest.

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